NON-EQUILIBRIUM GLAUBER TYPE DYNAMICS IN CONTINUUM

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Abstract

We construct the non-equilibrium Glauber dynamics as a Markov process in configuration space for an infinite particle system in continuum with a general class of initial distributions. This class we define in terms of correlation functions bounds and it is preserved during the Markov evolution we constructed.

Keywords: Configuration space, Glauber dynamics, non-equilibrium Markov process.

1 Introduction

The theory of stochastic lattice gases on the cubic lattice $\mathbb{Z}^d, d \in \mathbb{N}$ is one of the most important and well developed areas in the theory of interacting particle systems. In the lattice gas model with spin space $S = \{0, 1\}$, the configuration space is defined as $\mathcal{X} = \{0, 1\}^{\mathbb{Z}^d}$. For a given $\sigma = \{\sigma(x) \,|\, x \in \mathbb{Z}^d\} \in \mathcal{X}$ we say that a lattice side $y \in \mathbb{Z}^d$ is free or occupied by a particle depending on $\sigma(y) = 0$ or $\sigma(y) = 1$ respectively.

In the Glauber type stochastic dynamics of the lattice gas particles randomly disappear from occupied sites or appear of free places of the lattice. Obviously, this dynamics may be interpreted as a birth-and-death process on $\mathbb{Z}^d$. The generator of this dynamics is given by

$$(Lf)(\sigma) = \sum_{x \in \mathbb{Z}^d} a(x, \sigma)(\nabla_x f)(\sigma),$$

where

$$(\nabla_x f)(\sigma) = f(\sigma^x) - f(\sigma),$$

σ^x denoting the configuration σ in which the particle at site x has changed its spin value. The rate function $a(x, \sigma)$ is taken in such a way that an a priori given measure on $\mathcal{X}$ (say, a Gibbs measure for the Ising model) be a symmetrizing measure for the Glauber generator L, see, e.g., [17].

Let us consider a continuous particle system, i.e., a system of particles which can take any position in the Euclidean space $\mathbb{R}^d$. The configuration space Γ for such system is the set of all locally finite subsets $\gamma \subset \mathbb{R}^d$. An analog of the discussed lattice stochastic dynamics should be a process in which particles randomly appear and disappear in the space, i.e., a spatial birth-and-death process. The generator of such a process is informally given by the formula

$$(LF)(\gamma) = \sum_{x \in \gamma} d(x, \gamma)(D_x^- F)(\gamma) + \int_{\mathbb{R}^d} b(x, \gamma)(D_x^+ F)(\gamma)\, dx,$$

where

$$(D_x^- F)(\gamma) = F(\gamma \setminus x) - F(\gamma), \quad (D_x^+ F)(\gamma) = F(\gamma \cup x) - F(\gamma).$$

Here and below, for simplicity of notations, we just write x instead of $\{x\}$. The coefficient $d(x, \gamma)$ describes the rate at which the particle x of the configuration γ dies, while $b(x, \gamma)$ describes the rate at which, given the configuration γ, a new particle is born at x.

Spatial birth-and-death processes were first discussed by Preston in [19]. Under some conditions on the birth and death rates, Preston proved the existence of such processes in a bounded domain in $\mathbb{R}^d$. Though the number of particles can be arbitrarily large in this case, the total number of particles remains finite at any moment of time. The study of the problem of construction of a spatial birth-and-death process in the infinite volume was initiated by Holley and Stroock in [5]. In fact, in that paper, birth-and-death processes in bounded domains were analyzed in detail. Only in a very special case of nearest neighbor birth-and-death processes on the real line, the existence of a corresponding process on the whole space was proved and its properties were studied.

Glötzl [4] derived conditions on the coefficients $d(x,\gamma)$, $b(x,\gamma)$, under which the birth-and-death generators are symmetric in the space $L^2(\mu)$, where μ is a given Gibbs measure. Such generators is natural to call the Glauber dynamics generators (corresponding to the equilibrium state μ). However, the problem of existence of such dynamics was left open. In the paper [2], Bertini, Cancrini, and Cesi studied the problem of existence of a spectral gap for the Glauber dynamics in a bounded domain in $\mathbb{R}^d$. Bertini *et al.* considered the Glauber dynamics with death coefficient $d(x,\gamma) = 1$.

By using the theory of Dirichlet forms, an analog of the Glauber dynamics from [2], but on the whole space (thus, involving infinite configurations) and for quite general pair potentials, has been constructed in [11]. A general class of Glauber dynamics in continuum which admits much more wide family of birth and death rates (again in the framework of the Dirichlet forms theory) was considered in [12].

All mentioned papers are dealing with so-called equilibrium stochastic dynamics that gives an existence result for a.a. starting configurations w.r.t. the a priori given stationary measure. The latter means that we can start our Markov process with any initial measure which is absolutely continuous w.r.t. the symmetrizing one. In applications, however, we need to analyze the time development for different classes of initial states of the system. These states can be very far from the equilibrium ones and the equilibrium stochastic dynamics (coming from the Dirichlet forms method) is not enough for the construction of their evolution.

In the present paper we propose a construction of the non-equilibrium Glauber dynamics in continuum. Namely, we describe a set of initial distributions on Γ s.t. for any initial measure μ_0 from this class there exists a Markov process with considered Glauber generator $X_t^{\mu_0} \in \Gamma$ starting with μ_0. Moreover, the distribution μ_t of this process at the time $t > 0$ is again in the same class of measures on Γ. Our construction is based on a general approach to the study of infinite particle dynamics using techniques of the harmonic analysis on configuration spaces developed in [8]. More precisely, we start with the Kolmogorov equation corresponding to our Glauber dynamics. That is an evolutional equation on functions defined on the configuration space Γ which are called observables in the terminology of mathematical physics. An application of the combinatorial Fourier transform from [8] gives instead of this infinite dimensional evolution equation an infinite family of finite dimensional equations for so-called quasi-observables. This infinite system of equations admits a natural description in terms of a Fock-type structure. This structure is nothing but a L^1-Fock space with a fixed family of weight functions. Taking properly these weights we are able to apply a perturbation techniques to the considered evolution equation in the L^1-Fock space and to construct a related semigroup. The dual semigroup gives then the time evolution of the correlation functions of the initial measure and due to a reconstruction theorem from [8] we can obtain an evolution of the initial measure. The latter solve the dual Kolmogorov equation and it is the main step in the construction of the non-equilibrium Glauber dynamics we are considering.

Note that the evolution of the correlation functions in the Glauber dynamics is describing by a system of equations which give a dynamical version of the celebrated Kirkwood-Salsburg system of equations for an equilibrium state of the model. Solving this system we need to check a property of a positive definiteness for the solution in sense of [1], [8]. This moment is usually outside of the attention in theoretical physics considerations of time evolutions for correlation functions. But this positive definiteness is a necessary (and together with a growth condition also sufficient) condition on correlation functions which relates them to a measure on Γ. Actually, the verification of this condition is one of the main difficulties in the approach described above.

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2 General facts and notations

Let $\mathbb{R}^d$ be the d-dimensional Euclidean space. By $\mathcal{O}(\mathbb{R}^d)$, $\mathcal{B}(\mathbb{R}^d)$ we denote the family of all open and Borel sets, respectively. $\mathcal{O}_b(\mathbb{R}^d)$, $\mathcal{B}_b(\mathbb{R}^d)$ denote the system of all sets in $\mathcal{O}(\mathbb{R}^d)$, $\mathcal{B}(\mathbb{R}^d)$, respectively, which are bounded. The space of n-point configuration is

$$\Gamma_0^{(n)} = \Gamma_{0,\mathbb{R}^d}^{(n)} := \left\{ \eta \subset \mathbb{R}^d \middle| \, |\eta| = n \right\}, \quad n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\},$$

where $|A|$ denotes the cardinality of the set A. Analogously the space $\Gamma_{0,\Lambda}^{(n)}$ is defined for $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$, which we denote for short by $\Gamma_\Lambda^{(n)}$.

For every $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ one can define a mapping $N_\Lambda : \Gamma_0^{(n)} \to \mathbb{N}_0$; $N_\Lambda(\eta) := |\eta \cap \Lambda|$. For short we write $\eta_\Lambda := \eta \cap \Lambda$. As a set, $\Gamma_0^{(n)}$ is equivalent to the symmetrization of

$$\widetilde{(\mathbb{R}^d)^n} = \left\{ (x_1, \ldots, x_n) \in (\mathbb{R}^d)^n \middle| \, x_k \neq x_l \text{ if } k \neq l \right\},$$

i.e. $\widetilde{(\mathbb{R}^d)^n}/S_n$, where S_n is the permutation group over $\{1, \ldots, n\}$. Hence $\Gamma_0^{(n)}$ inherits the structure of an $n \cdot d$-dimensional manifold. Applying this we can introduce a topology $\mathcal{O}(\Gamma_0^{(n)})$ on $\Gamma_0^{(n)}$. The corresponding Borel σ-algebra $\mathcal{B}(\Gamma_0^{(n)})$ coincides with $\sigma\left(N_\Lambda \,\middle|\, \Lambda \in \mathcal{B}_b(\mathbb{R}^d)\right)$.

The space of finite configurations

$$\Gamma_0 := \bigsqcup_{n \in \mathbb{N}_0} \Gamma_0^{(n)}$$

is equipped with the topology of disjoint union $\mathcal{O}(\Gamma_0)$. A set $B \in \mathcal{B}(\Gamma_0)$ (the corresponding Borel σ-algebra) is called bounded if there exists a $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ and an $N \in \mathbb{N}$ such that $B \subset \bigsqcup_{n=0}^N \Gamma_\Lambda^{(n)}$.

The configuration space

$$\Gamma := \left\{ \gamma \subset \mathbb{R}^d \middle| \, |\gamma \cap \Lambda| < \infty, \text{ for all } \Lambda \in \mathcal{B}_b(\mathbb{R}^d) \right\}$$

is equipped with the vague topology. The Borel σ-algebra $\mathcal{B}(\Gamma)$ is equal to the smallest σ-algebra for which all the mappings $N_\Lambda : \Gamma \to \mathbb{N}_0$, $N_\Lambda(\gamma) := |\gamma \cap \Lambda|$ are measurable, i.e.,

$$\mathcal{B}(\Gamma) = \sigma\left(N_\Lambda \,\middle|\, \Lambda \in \mathcal{B}_b(\mathbb{R}^d)\right)$$

and filtration on Γ given by

$$\mathcal{B}_\Lambda(\Gamma) := \sigma\left(N_{\Lambda'} \,\middle|\, \Lambda' \in \mathcal{B}_b(\mathbb{R}^d),\ \Lambda' \subset \Lambda\right).$$

For every $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ one can define a projection $p_\Lambda : \Gamma \to \Gamma_\Lambda$,

$$p_\Lambda(\gamma) := \gamma_\Lambda$$

and w.r.t. this projections Γ is the projective limit of the spaces $\{\Gamma_\Lambda\}_{\Lambda \in \mathcal{B}_b(\mathbb{R}^d)}$.

In the sequel we will use the following classes of function: $L^0(\Gamma_0)$ is the set of all measurable functions on Γ_0 and $L^0_{\mathrm{ls}}(\Gamma_0)$ is the set of measurable functions which have additionally a local support, i.e. $G \in L^0_{\text{ls}}(\Gamma_0)$ if there exists $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ such that $G \restriction_{\Gamma_0 \setminus \Gamma_\Lambda} = 0$. $L^0_{\text{bs}}(\Gamma_0)$ denotes the class of measurable functions with bounded support, $B(\Gamma_0)$ the set of bounded measurable functions and $B_{\text{bs}}(\Gamma_0)$ the set of bounded functions with bounded support. For any $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$, the class of functions $G \in B_{\text{bs}}(\Gamma_0)$, whose support is a subset of Λ we will denote by $B^\Lambda_{\text{bs}}(\Gamma_0)$. The class of continuous functions from $B^\Lambda_{\text{bs}}(\Gamma_0)$ we will denote by $CB^\Lambda_{\text{bs}}(\Gamma_0)$.

On Γ we consider the set of a cylinder functions $\mathcal{F}L^0(\Gamma)$, i.e. the set of all measurable function $G \in L^0(\Gamma)$ which are measurable w.r.t. $\mathcal{B}_\Lambda(\Gamma)$ for some $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$. These functions are characterized by the following relation: $F(\gamma) = F \restriction_{\Gamma_\Lambda} (\gamma_\Lambda)$.

Next we would like to describe some facts from Harmonic analysis on configuration space based on [8, 9].

The following mapping between functions on Γ_0, e.g. $L^0_{\text{ls}}(\Gamma_0)$, and functions on Γ, e.g. $\mathcal{F}L^0(\Gamma)$, plays a key role in our further considerations:

$$KG(\gamma) := \sum_{\xi \Subset \gamma} G(\xi), \quad \gamma \in \Gamma,$$

where $G \in L^0_{\text{ls}}(\Gamma_0)$, see e.g. [15, 16]. The summation in the latter expression is extend over all finite subconfigurations of γ, in symbols $\xi \Subset \gamma$.

K is linear, positivity preserving, and invertible, with

$$K^{-1}F(\eta) := \sum_{\xi \subset \eta} (-1)^{|\eta \setminus \xi|} F(\xi), \quad \eta \in \Gamma_0. \tag{1}$$

It is easy to see that for all $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$, $F \in \mathcal{F}L^0(\Gamma,\ \mathcal{B}_\Lambda(\Gamma))$

$$K^{-1}F(\eta) = 1\!\!1_{\Gamma_\Lambda}(\eta) K^{-1}F(\eta), \quad \forall \eta \in \Gamma_0. \tag{2}$$

One can introduce a convolution

$$\begin{aligned} \star : L^0(\Gamma_0) \times L^0(\Gamma_0) &\rightarrow L^0(\Gamma_0) \\ (G_1, G_2) &\mapsto (G_1 \star G_2)(\eta) \\ &:= \sum_{(\xi_1, \xi_2, \xi_3) \in \mathcal{P}^3_\emptyset(\eta)} G_1(\xi_1 \cup \xi_2)\ G_2(\xi_2 \cup \xi_3), \end{aligned} \tag{3}$$

where $\mathcal{P}^3_\emptyset(\eta)$ denotes the set of all partitions (ξ_1, ξ_2, ξ_3) of η in 3 parts, i.e., all triples (ξ_1, ξ_2, ξ_3) with $\xi_i \subset \eta$, $\xi_i \cap \xi_j = \emptyset$ if $i \neq j$, and $\xi_1 \cup \xi_2 \cup \xi_3 = \eta$. It has the property that for $G_1, G_2 \in L^0_{\text{ls}}(\Gamma_0)$ we have $K(G_1 \star G_2) = KG_1 \cdot KG_2$. Due to this convolution we can interpret K transform as Fourier transform in configuration space analysis, see also [1].

Let $\mathcal{M}^1_{\text{fm}}(\Gamma)$ be the set of all probability measures μ which have finite local moments of all orders, i.e. $\int_\Gamma |\gamma_\Lambda|^n \mu(d\gamma) < +\infty$ for all $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$

and $n \in \mathbb{N}_0$. A measure ρ on Γ_0 is called locally finite iff $\rho(A) < \infty$ for all bounded sets A from $\mathcal{B}(\Gamma_0)$, the set of such measures is denoted by $\mathcal{M}_{\mathrm{lf}}(\Gamma_0)$. One can define a transform $K^* : \mathcal{M}^1_{\mathrm{fm}}(\Gamma) \to \mathcal{M}_{\mathrm{lf}}(\Gamma_0)$, which is dual to the K-transform, i.e., for every $\mu \in \mathcal{M}^1_{\mathrm{fm}}(\Gamma)$, $G \in \mathcal{B}_{\mathrm{bs}}(\Gamma_0)$ we have

$$\int_\Gamma KG(\gamma)\mu(d\gamma) = \int_{\Gamma_0} G(\eta)\,(K^*\mu)(d\eta).$$

$\rho_\mu := K^*\mu$ we call the correlation measure corresponding to μ.

As shown in [8] for $\mu \in \mathcal{M}^1_{\mathrm{fm}}(\Gamma)$ and any $G \in L^1(\Gamma_0, \rho_\mu)$ the series

$$KG(\gamma) := \sum_{\eta \Subset \gamma} G(\eta), \tag{4}$$

is μ-a.s. absolutely convergent. Furthermore, $KG \in L^1(\Gamma, \mu)$ and

$$\int_{\Gamma_0} G(\eta)\,\rho_\mu(d\eta) = \int_\Gamma (KG)(\gamma)\,\mu(d\gamma). \tag{5}$$

Fix a non-atomic and locally finite measure σ on $(\mathbb{R}^d, \mathcal{B}(\mathbb{R}^d))$. For any $n \in \mathbb{N}$ the product measure $\sigma^{\otimes n}$ can be considered by restriction as a measure on $\widetilde{(\mathbb{R}^d)^n}$ and hence on $\Gamma_0^{(n)}$. The measure on $\Gamma_0^{(n)}$ we denote by $\sigma^{(n)}$.

The *Lebesgue-Poisson measure* $\lambda_{z\sigma}$ on Γ_0 is defined as

$$\lambda_{z\sigma} := \sum_{n=0}^{\infty} \frac{z^n}{n!}\sigma^{(n)}.$$

Here $z > 0$ is the so called activity parameter. The restriction of $\lambda_{z\sigma}$ to Γ_Λ will be also denoted by $\lambda_{z\sigma}$.

The *Poisson measure* $\pi_{z\sigma}$ on $(\Gamma, \mathcal{B}(\Gamma))$ is given as the projective limit of the family of measures $\{\pi^\Lambda_{z\sigma}\}_{\Lambda \in \mathcal{B}_b(\mathbb{R}^d)}$, where $\pi^\Lambda_{z\sigma}$ is the measure on Γ_Λ defined by $\pi^\Lambda_{z\sigma} := e^{-z\sigma(\Lambda)}\lambda_{z\sigma}$.

A measure $\mu \in \mathcal{M}^1_{\mathrm{fm}}(\Gamma)$ is called locally absolutely continuous w.r.t. $\pi_{z\sigma}$ iff $\mu_\Lambda := \mu \circ p_\Lambda^{-1}$ is absolutely continuous with respect to $\pi^\Lambda_{z\sigma} = \pi_{z\sigma} \circ p_\Lambda^{-1}$ for all $\Lambda \in \mathcal{B}_\Lambda(\mathbb{R}^d)$. In this case $\rho_\mu := K^*\mu$ is absolutely continuous w.r.t $\lambda_{z\sigma}$. We denote by $k_\mu(\eta) := \frac{d\rho_\mu}{d\lambda_\sigma}(\eta)$, $\eta \in \Gamma_0$.

The functions

$$k_\mu^{(n)} : (\mathbb{R}^d)^n \longrightarrow \mathbb{R}_+ \tag{6}$$

$$k_\mu^{(n)}(x_1, \ldots, x_n) := \begin{cases} k_\mu(\{x_1, \ldots, x_n\}), & \text{if } (x_1, \ldots, x_n) \in \widetilde{(\mathbb{R}^d)^n} \\ 0, & \text{otherwise} \end{cases}$$

are well known correlation functions of statistical physics, see e.g [22], [21].

Let us now recall the so-called Minlos lemma which plays very important role in our calculations (cf., [13]).

Lemma 2.1 *Let $n \in \mathbb{N}$, $n \geq 2$, and $z > 0$ be given. Then*

$$\int_{\Gamma_0} \ldots \int_{\Gamma_0} G(\eta_1 \cup \ldots \cup \eta_n) H(\eta_1, \ldots, \eta_n) d\lambda_z(\eta_1) \ldots d\lambda_z(\eta_n) =$$

$$= \int_{\Gamma_0} G(\eta) \sum_{(\eta_1,\ldots,\eta_n) \in \mathcal{P}_n(\eta)} H(\eta_1, \ldots, \eta_n) d\lambda_z(\eta)$$

for all measurable functions $G : \Gamma_0 \mapsto \mathbb{R}$ and $H : \Gamma_0 \times \ldots \times \Gamma_0 \mapsto \mathbb{R}$ with respect to which both sides of the equality make sense. Here $\mathcal{P}_n(\eta)$ denotes the set of all partitions of η in n parts, which may be empty.

3 Potential and Gibbs measures on configuration spaces

A pair potential is a Borel, even function $\phi : \mathbb{R}^d \mapsto \mathbb{R} \cup \{+\infty\}$. We assume that ϕ satisfies the following conditions.

(I) (Integrability) For any $\beta > 0$,

$$C(\beta) := \int_{\mathbb{R}^d} |1 - \exp[-\beta\phi(x)]| dx < \infty.$$

(P) (Positivity) $\phi(x) > 0$ for all $x \in \mathbb{R}^d$.

For $\gamma \in \Gamma$ and $x \in \mathbb{R}^d \setminus \gamma$, we define a relative energy of interaction as follows:

$$E(x, \gamma) := \begin{cases} \sum_{y \in \gamma} \phi(x - y), & \text{if } \sum_{y \in \gamma} |\phi(x - y)| < \infty, \\ +\infty, & \text{otherwise.} \end{cases}$$

The energy of configuration $\eta \in \Gamma_0$ or Hamiltonian $E^\phi : \Gamma_0 \to \mathbb{R} \cup \{+\infty\}$ which corresponds to potential ϕ is defined by

$$E^\phi(\eta) = \sum_{\{x,y\} \subset \eta} \phi(x - y), \ \eta \in \Gamma_0, \ |\eta| \geq 2.$$

The Hamiltonian $E^\phi_\Lambda : \Gamma_\Lambda \to \mathbb{R}$ for $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ which corresponds to potential ϕ is defined by

$$E^\phi_\Lambda(\eta) = \sum_{\{x,y\} \subset \eta} \phi(x - y), \ \eta \in \Gamma_\Lambda, \ |\eta| \geq 2.$$

For fixed ϕ we will write for short $E = E^\phi$ and $E_\Lambda = E^\phi_\Lambda$.

For given $\bar{\gamma} \in \Gamma$ define the interaction energy between $\eta \in \Gamma_\Lambda$ and $\bar{\gamma}_{\Lambda^c} = \bar{\gamma} \cap \Lambda^c$, $\Lambda^c = \mathbb{R}^d \backslash \Lambda$ as

$$W_\Lambda(\eta|\bar{\gamma}) = \sum_{x \in \eta,\ y \in \bar{\gamma}_{\Lambda^c}} \phi(x-y).$$

The interaction energy is said to be well-defined if for any $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$, $\eta \in \Gamma_\Lambda$ and $\bar{\gamma} \in \Gamma$ it is finite or $+\infty$.

For $\beta > 0$ we define

$$E_\Lambda(\eta|\bar{\gamma}) = E_\Lambda(\eta) + W_\Lambda(\eta|\bar{\gamma})$$

and

$$Z_\Lambda(\bar{\gamma}) := \int_{\Gamma_\Lambda} \exp\{-\beta E_\Lambda(\eta|\ \bar{\gamma})\} \lambda_z(d\eta)$$

the so-called partition function.

Let $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$, $\beta > 0$ be arbitrary, and let $\bar{\gamma} \in \Gamma$. The finite volume Gibbs measure on the space Γ_Λ with boundary configuration $\bar{\gamma}$ is defined by

$$\mathrm{P}_{\Lambda,\bar{\gamma}}(d\eta) = \frac{\exp\{-\beta E_\Lambda(\eta|\ \bar{\gamma})\}}{Z_\Lambda(\bar{\gamma})} \lambda_z(d\eta).$$

When $\bar{\gamma} = \emptyset$, let $\mathrm{P}_{\Lambda,\emptyset}(d\eta) =: \mathrm{P}_\Lambda(d\eta)$.

Let $\{\pi_\Lambda\}$ denote the specification associated with z and the Hamiltonian E (see [18]) which is defined by

$$\pi_{\Lambda,\bar{\gamma}}(A) = \int_{A'} P_{\Lambda,\bar{\gamma}}(d\eta)$$

where $A' = \{\eta \in \Gamma_\Lambda :\ \eta \cup (\bar{\gamma}_{\Lambda^c}) \in A\}$, $A \in \mathcal{B}(\Gamma)$ and $\bar{\gamma} \in \Gamma$.

A probability measure μ on Γ is called a Gibbs measure for E and z if

$$\mu(\pi_{\Lambda,\bar{\gamma}}(A)) = \mu(A)$$

for every $A \in \mathcal{B}(\Gamma)$ and every $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$.

This relation is the well-known (DLR)-equation (Dobrushin-Lanford-Ruelle equation), see [3] for more details.

The set of all Gibbs measures which corresponds to the potential ϕ, activity parameter $z > 0$, and inverse temperature $\beta > 0$ will be denoted by $\mathcal{G}(\phi, z, \beta)$. For fixed potential ϕ we will write $\mathcal{G}(z, \beta)$ instead of $\mathcal{G}(\phi, z, \beta)$.

Remark 3.1 *The set $\mathcal{G}(\phi, z, \beta)$ is non-empty for all $z > 0$, $\beta > 0$ and any potential ϕ satisfying* **(P)** *and* **(I)***, see [14].*

4 Generators. The symbol of the Glauber generator on the space of finite configurations

We consider a Markov pre-generator on the configuration space Γ, the action of which is given by

$$(LF)(\gamma) := (L_{b,d})F(\gamma) =$$
$$= \sum_{x\in\gamma} d(x,\gamma\setminus x)D_x^- F(\gamma) + \int_{\mathbb{R}^d} b(x,\gamma)D_x^+ F(\gamma)dx,$$

where $D_x^- F(\gamma) = F(\gamma\setminus x) - F(\gamma)$ and $D_x^+ F(\gamma) = F(\gamma\cup x) - F(\gamma)$.

It is known that the Gibbs measure $\mu \in \mathcal{G}(z,\,\beta)$ is reversible with respect to the Markov process associated with the generator L (i.e. the operator L is symmetrical in $L^2(\Gamma,\,\mu)$) iff the following condition on coefficients b and d (birth and death rates) holds:

$$b(x,\,\gamma) = ze^{-\beta E(x,\,\gamma)}d(x,\,\gamma). \tag{7}$$

In the sequel we will be interesting only in particular cases of birth and death rates, which play an essential role in the study of some problems of mathematical physics:

$$b(x,\,\gamma) = ze^{-\beta E(x,\,\gamma)},\quad d(x,\,\gamma) = 1.$$

Such model was investigated by many authors, see e.g. [13], [11]. The corresponding Markov generator we denote by the same symbol L.

For the technical reasons we will be also interesting in the birth and death rates localized in some volume $\Lambda \in \mathcal{B}(\mathbb{R}^d)$. Namely

$$b_\Lambda(x,\,\gamma) = z1\!\!1_\Lambda(x)e^{-\beta E(x,\,\gamma_\Lambda)},\quad d_\Lambda(x,\,\gamma) = 1\!\!1_\Lambda(x).$$

Corresponding Markov generator we denote by L_Λ.

In the recent paper [13], the authors have shown that in the case of equilibrium Glauber dynamics with invariant measure $\mu \in \mathcal{G}(z,\,\beta)$, corresponding to the pair potential, the image of L under the K-transform (or symbol of the operator L) has the following form:

$$(\widehat{L}G)(\eta) := (K^{-1}LKG)(\eta) = L_0G(\eta) + L_1G(\eta),\;\; G \in B_{\mathrm{bs}}(\Gamma_0)$$

where

$$L_0G(\eta) := -|\eta|G(\eta)$$

and

$$L_1G(\eta) := z\sum_{\xi\subseteq\eta}\int_{\mathbb{R}^d} G(\xi\cup x)\prod_{y\in\eta\setminus\xi}(e^{-\beta\phi(x-y)}-1)\prod_{y'\in\xi}e^{-\beta\phi(x-y')}dx.$$

Analogously, one can show that the symbol of L_Λ has form:

$$(\widehat{L_\Lambda}G)(\eta) := (K^{-1}L_\Lambda KG)(\eta) = -|\eta_\Lambda|G(\eta)+$$
$$+z\sum_{\xi\subseteq\eta}\int_\Lambda G(\xi\cup x)\prod_{y\in\eta\setminus\xi}(e^{-\beta\phi(x-y)1\!\!1_\Lambda(y)}-1)\prod_{y'\in\xi_\Lambda}e^{-\beta\phi(x-y')}dx,\ \ G\in B_{\mathrm{bs}}(\Gamma_0).$$

5 Construction of a semigroup of the symbol.

Let λ be the Lebesgue-Poisson measure on Γ_0 with activity parameter equal to 1. In the whole section we suppose that potential ϕ satisfies condition **(P)** and **(I)**.

For arbitrary and fixed $C>0$ and $\beta>0$, we consider operator $\widehat{L}$ as a pre-generator of some non-equilibrium Markov process in the functional space

$$\mathcal{L}_{C,\beta} := L^1(\Gamma_0, C^{|\eta|}e^{-\beta E(\eta)}\lambda(d\eta)). \tag{8}$$

In this section, symbol $||\cdot||$ stands for the norm of the space (8) and symbol $\stackrel{s}{\to}$ denote strong convergence in $\mathcal{L}_{C,\beta}$.

For any $\Lambda\in\mathcal{B}_b(\mathbb{R}^d)$ we set

$$\mathcal{L}^\Lambda_{C,\beta} := \left\{G\in\mathcal{L}_{C,\beta}\ |\ G\upharpoonright_{\Gamma_0\setminus\Gamma_\Lambda}=0\right\}. \tag{9}$$

It is not difficult to show that $\mathcal{L}^\Lambda_{C,\beta}$ is a closed linear subset in $(\mathcal{L}_{C,\beta}\,,\,||\cdot||)$. Therefore, $(\mathcal{L}^\Lambda_{C,\beta}\,,\,||\cdot||)$ is a subspace of $(\mathcal{L}_{C,\beta}\,,\,||\cdot||)$.

For any $\omega>0$ we introduce a set $\mathcal{H}(\omega,0)$ of all densely defined closed operators T on $\mathcal{L}_{C,\beta}$, the resolvent set $\rho(T)$ of which contains sector

$$Sect\left(\frac{\pi}{2}+\omega\right) := \left\{\zeta\in\mathbb{C}\ |\ |arg\,\zeta|<\frac{\pi}{2}+\omega\right\},\quad \omega>0$$

and for any $\varepsilon>0$

$$||(T-\zeta1\!\!1)^{-1}||\le\frac{M_\varepsilon}{|\zeta|},\quad |arg\,\zeta|\le\frac{\pi}{2}+\omega-\varepsilon,$$

where M_ε does not depend on ζ.

Let $\mathcal{H}(\omega,\theta)$, $\theta\in\mathbb{R}$ denote the set of all operators of the form $T=T_0+\theta$ with $T_0\in\mathcal{H}(\omega,0)$.

Remark 5.1 *It is well-known (see e.g., [7]), that any $T\in\mathcal{H}(\omega,\theta)$ is a generator of a semigroup $U(t)$ which is holomorphic in the sector $|arg\,t|<\omega$. The function $U(t)$ is not necessary uniformly bounded, but it is quasi-bounded, i.e.*

$$||U(t)||\le const\,|e^{\theta t}|$$

in any sector of the form $|arg\,t|\le\omega-\varepsilon$.

Proposition 5.1 *For any $C > 0$ and $\beta > 0$, the operator*

$$(L_0G)(\eta) = -|\eta|G(\eta), \quad D(L_0) = \{G \in \mathcal{L}_{C,\beta} \mid |\eta|G(\eta) \in \mathcal{L}_{C,\beta}\}$$

is a generator of contraction semigroup on $\mathcal{L}_{C,\beta}$. Moreover, $L_0 \in \mathcal{H}(\omega, 0)$, for all $\omega \in (0, \frac{\pi}{2})$.

Proof. It is not difficult to show that L_0 is a densely defined and closed operator in $\mathcal{L}_{C,\beta}$.

Let $0 < \omega < \frac{\pi}{2}$ be arbitrary and fixed. Clear, that for all $\zeta \in Sect\left(\frac{\pi}{2} + \omega\right)$

$$||\eta| + \zeta| > 0, \quad \eta \in \Gamma_0.$$

Therefore, for any $\zeta \in Sect\left(\frac{\pi}{2} + \omega\right)$ the inverse operator $(L_0 - \zeta 1\!\!1)^{-1}$, the action of which is given by

$$[(L_0 - \zeta 1\!\!1)^{-1}G](\eta) = -\frac{1}{|\eta| + \zeta}\, G(\eta), \tag{10}$$

is well defined on the whole space $\mathcal{L}_{C,\beta}$. Moreover, it is bounded operator in this space and

$$||(L_0 - \zeta 1\!\!1)^{-1}|| \le \begin{cases} \frac{1}{|\zeta|}, & \text{if } Re\,\zeta \ge 0, \\ \\ \frac{M}{|\zeta|}, & \text{if } Re\,\zeta < 0, \end{cases} \tag{11}$$

where constant M does not depend on ζ.

The case $Re\,\zeta \ge 0$ is a direct consequence of (10) and inequality

$$|\eta| + Re\,\zeta \ge Re\;\zeta \ge 0.$$

We prove now bound (11) in the case $Re\,\zeta < 0$. Using (10), we have

$$||(L_0 - \zeta 1\!\!1)^{-1}G|| = \left|\left|\frac{1}{|| \cdot | + \zeta|}\, G(\cdot)\right|\right| = \frac{1}{|\zeta|}\left|\left|\frac{|\zeta|}{|| \cdot | + \zeta|} G(\cdot)\right|\right|.$$

Since $\zeta \in Sect\left(\frac{\pi}{2} + \omega\right)$,

$$|Im\,\zeta| \ge |\zeta| \left|\sin\left(\frac{\pi}{2} + \omega\right)\right| = |\zeta| \cos\omega.$$

Hence,

$$\frac{|\zeta|}{||\eta| + \zeta|} \le \frac{|\zeta|}{|Im\,\zeta|} \le \frac{1}{\cos\omega} =: M$$

and (11) is fulfilled.

The rest statement of the lemma follows now directly from the theorem of Hille-Iosida (see e.g., [7]). ■

Let $\kappa > 0$ be the parameter of the considering model.

We now consider operator

$$(L_1 G)(\eta) = (L_{1,\,\beta,\,\kappa} G)(\eta) =$$

$$= \kappa \sum_{\xi\subseteq\eta} \int_{\mathbb{R}^d} G(\xi\cup x) \prod_{y\in\eta\setminus\xi} (e^{-\beta\phi(x-y)} - 1)\, e^{-\beta E(x,\,\xi)} dx, \quad G \in D(L_1)$$

with domain $D(L_1) := D(L_0)$. The well-definiteness of this operator will be clear from the lemma below.

For the symbol of the operator L we will write sometimes $\widehat{L_{\beta,\,\kappa}}$ instead of $\widehat{L}$ to stress the dependence of this operator on $\kappa > 0$ and $\beta > 0$.

Lemma 5.1 *For any $\delta > 0$ there exists $\kappa_0 := \kappa_0(\delta) > 0$ such that for all $\kappa < \kappa_0$ the following estimate holds*

$$||L_{1,\,\beta,\,\kappa} G|| \leq a||L_0 G|| + b||G||, \quad G \in D(L_0) = D(L_1), \tag{12}$$

with $a = a(\kappa) < \delta$ and $b = b(\kappa) < \delta$.

Proof. As 1 belong to the resolvent set of L_0 we have

$$||L_1(L_0 - 1\!\!1)^{-1} G|| = \kappa \int_{\Gamma_0} \left| L_1(L_0 - 1\!\!1)^{-1} G(\eta) \right| C^{|\eta|} e^{-\beta E(\eta)} \lambda(d\eta). \tag{13}$$

Define

$$K(x,\eta) := \prod_{y\in\eta\setminus x} |e^{-\beta\phi(x-y)} - 1|, \quad x \in \mathbb{R}^d, \quad \eta \in \Gamma_0,$$

then by modulus property (13) can be estimated by

$$\kappa \int_{\Gamma_0} \sum_{\xi\subseteq\eta} \int_{\mathbb{R}^d} \frac{1}{|\xi\cup x| + 1} |G(\xi\cup x)|\, K(x, \eta\setminus\xi)\, e^{-\beta E(x,\,\xi)} dx\, C^{|\eta|} e^{-\beta E(\eta)} \lambda(d\eta). \tag{14}$$

By Minlos lemma, (14) is equal to

$$\kappa \int_{\Gamma_0} \int_{\Gamma_0} \int_{\mathbb{R}^d} \frac{1}{|\xi\cup x| + 1} |G(\xi\cup x)|\, K(x,\eta) e^{-\beta E(x,\,\xi)} dx\, C^{|\eta\cup\xi|} e^{-\beta E(\eta\cup\xi)} \lambda(d\xi)\lambda(d\eta).$$

Using again Minlos lemma we bound the latter expression by

$$\kappa \int_{\Gamma_0} \int_{\Gamma_0} \frac{1}{|\xi| + 1} |G(\xi)| \sum_{x\in\xi} K(x,\eta) e^{-\beta E(x,\,\xi\setminus x)} C^{|\eta\cup(\xi\setminus x)|} e^{-\beta E(\eta\cup(\xi\setminus x))} \lambda(d\xi)\lambda(d\eta).$$

Since

$$E(x,\,\xi\setminus x)=E(\xi)-E(\xi\setminus x)$$

and since positivity of ϕ implies

$$E(\eta\cup(\xi\setminus x))-E(\xi\setminus x)\geq 0$$

we have

$$||L_1(L_0-1\!\!1)^{-1}G||\leq$$

$$\leq\kappa\int_{\Gamma_0}\frac{1}{|\xi|+1}|G(\xi)|\,C^{|\xi|}e^{-\beta E(\xi)}\sum_{x\in\xi}\int_{\Gamma_0}K(x,\eta)C^{|\eta\cup(\xi\setminus x)|-|\xi|}\lambda(d\eta)\lambda(d\xi)\leq$$

$$\leq\kappa\int_{\Gamma_0}\frac{1}{|\xi|+1}|G(\xi)|\,C^{|\xi|}e^{-\beta E(\xi)}\sum_{x\in\xi}\int_{\Gamma_0}K(x,\eta)C^{|\eta|-1}\lambda(d\eta)\lambda(d\xi).$$

Finally,

$$||L_1(L_0-1)^{-1}G||\leq\kappa C^{-1}e^{C(\beta)C}||G||.$$

Therefore,

$$||L_1G||\leq\kappa C^{-1}e^{C(\beta)C}||(L_0-1)G||\leq a||L_0G||+b||G||,$$

where

$$a=b:=\kappa C^{-1}e^{C(\beta)C}.$$

Clear, that taking

$$\kappa_0=\delta Ce^{-C(\beta)C}$$

we obtain that $a,\,b<\delta$. ■

Theorem 5.1 *For any $C>0$, and for all $\kappa,\,\beta>0$ which satisfy*

$$2\kappa\exp\left(C(\beta)C\right)<C, \tag{15}$$

the operator $\widehat{L_{\beta,\,\kappa}}$ is a generator of a holomorphic semigroup in $\mathcal{L}_{C,\,\beta}$.

Proof. The statement of theorem follows directly from the theorem about perturbation of holomorphic semigroup (see, e.g. [7]). For the reader's convenience, below we give its formulation:

for any $T\in\mathcal{H}(\omega,\,\theta)$ and for any $\varepsilon>0$ there exists positive constants ϵ, δ such that if operator A satisfies

$$||Au||\leq a||Tu||+b||u||,\quad u\in D(T)\subset D(A),$$

with $a<\delta$, $b<\delta$, then $T+A\in\mathcal{H}(\omega-\varepsilon,\,\epsilon)$.

In particular, if $\theta=0$ and $b=0$, then $T+A\in\mathcal{H}(\omega-\varepsilon,\,0)$. ■

Remark 5.2 *Applying the proof of the theorem about perturbation of the generator of a holomorphic semigroup (see, e.g. [7]) to our case and taking into account the fact that* $L_0 \in \mathcal{H}(\omega, 0)$*, for any* $\omega \in (0, \frac{\pi}{2})$*, one can conclude that* δ *in this theorem can be chosen to be* $\frac{1}{2}$*.*

For our further purposes we have to show that holomorphic semigroup constructed in Theorem 5.1 can be approximated by the semigroups localized in bounded volumes.

Let $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ be arbitrary and fixed. Then all results proved in this section hold true for the operator $\widehat{L_\Lambda}$ acting in the functional space $\mathcal{L}^{\Lambda}_{C,\beta}$ with domain

$$D(\widehat{L_\Lambda}) := \{G \in \mathcal{L}_{C,\beta} \mid \ |\cdot_\Lambda| G(\cdot) \in \mathcal{L}^{\Lambda}_{C,\beta}\}.$$

Namely, the main result can be formulated as follows

Theorem 5.2 *For any* $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$*, and any triple of constants* C*,* $\kappa > 0$*, and* $\beta > 0$ *which satisfy*

$$2\kappa \exp\left(C(\beta)C\right) < C,$$

the operator $\widehat{L_\Lambda}$ *is a generator of a holomorphic semigroup in* $\mathcal{L}^{\Lambda}_{C,\beta}$*.*

Remark 5.3 *The arguments, analogous to those which were proposed in the proof of Lemma 5.1, imply the fulfilment of (12) for the operators*

$$\widehat{L_{0,\Lambda}} G(\eta) := |\eta_\Lambda| G(\eta)$$

and

$$\widehat{L_{1,\Lambda}} G(\eta) := \kappa \sum_{\xi \subseteq \eta} \int_\Lambda G(\xi \cup x) \prod_{y \in \eta \setminus \xi} (e^{-\beta\phi(x-y) 1\!\!1_\Lambda(y)} - 1) \prod_{y' \in \xi_\Lambda} e^{-\beta\phi(x-y')} dx$$

with

$$D(\widehat{L_{0,\Lambda}}) = D(\widehat{L_{1,\Lambda}}) := \{G \in \mathcal{L}_{C,\beta} \mid |\cdot_\Lambda| G(\cdot) \in \mathcal{L}^{\Lambda}_{C,\beta}\}.$$

Moreover, bound (12) in this case will be uniform with respect to the $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$*, i.e. coefficients* $a > 0$ *and* $b > 0$ *in (12) can be chosen* Λ *independent.*

Fix any triple of positive constants C, κ and β which satisfies (15) and any $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$.

Remark 5.4 *Let* $\widehat{U_t^\Lambda}(C,\beta,\kappa)$ *be holomorphic semigroup generated by operator* $\left(\widehat{L_\Lambda},\, D(\widehat{L_\Lambda})\right)$ *on* $\mathcal{L}_{C,\,\beta}^\Lambda$*. Then* $\widehat{U_t^\Lambda}(C,\beta,\kappa)\circ P_\Lambda,\ _{t\geq 0}$ *, where*

$$P_\Lambda G(\eta) := 1\!\!1_{\Gamma_\Lambda}(\eta)G(\eta), \quad G\in\mathcal{L}_{C,\,\beta}$$

is a semigroup on $\mathcal{L}_{C,\,\beta}$ *generated by the operator* $\widehat{L_\Lambda}\circ P_\Lambda$ *with domain*

$$D(\widehat{L_\Lambda}\circ P_\Lambda) := \{G\in\mathcal{L}_{C,\,\beta}\,|\;|\cdot_\Lambda|1\!\!1_{\Gamma_\Lambda}(\cdot)G(\cdot)\in\mathcal{L}_{C,\,\beta}\}\,.$$

Remark 5.5 *The theorem about perturbation of the generator of a holomorphic semigroup, mentioned before in this section (see also [7]), implies that for any* $\Lambda\in\mathcal{B}_b(\mathbb{R}^d)$ *and* $\varepsilon>0$ *there exists* $\epsilon>0$ *and constant* $M>0$ *which is not depend on* Λ *such that for any* $\zeta>\epsilon$ *the following bound holds*

$$||(\widehat{L_\Lambda}\circ P_\Lambda-\zeta)^{-1}||\leq\frac{M_\varepsilon}{|\zeta-\epsilon|},\quad |arg\,(\zeta-\epsilon)|\leq\frac{\pi}{2}+\omega-\varepsilon.$$

Let $\{\Lambda_n\}_{n\geq 1}$ be a sequence of Borel sets such that $\Lambda_n\subset\Lambda_{n+1},$ for all $n\in\mathbb{N}$, and $\bigcup_{n\geq 1}\Lambda_n=\mathbb{R}^d$. Now, we formulate the following approximation theorem.

Theorem 5.3 *Let* $\widehat{U_t}(C,\beta,\kappa)$ *and* $\left\{\widehat{U_t^{\Lambda_n}}(C,\beta,\kappa),\, n\geq 1\right\}$ *be holomorphic semigroups generated by* $\widehat{L_{\beta,\,\kappa}}$ *and* $\left\{\widehat{L_{\Lambda_n,\,\beta,\,\kappa}},\, n\geq 1\right\}$ *in the spaces* $\mathcal{L}_{C,\,\beta}$ *and* $\mathcal{L}_{C,\,\beta}^\Lambda$*, respectively. Then,*

$$\widehat{U_t^{\Lambda_n}}(C,\beta,\kappa)\circ P_{\Lambda_n}\overset{s}{\to}\widehat{U_t}(C,\beta,\kappa),\quad n\to\infty$$

uniformly on any finite interval of $t\geq 0$*.*

Proof. Using approximation theorem for quasi-bounded semigroups (see e.g. [7]), it is enough to show that

$$(\widehat{L_{\Lambda_n,\,\beta,\,\kappa}}\circ P_{\Lambda_n}-\zeta)^{-1}\overset{s}{\to}(\widehat{L_{\beta,\,\kappa}}-\zeta)^{-1}$$

for some $\zeta\in\mathbb{C}$ such that $\mathrm{Re}\,\zeta>\theta$.

Let $\zeta\in\mathbb{C}$, $\mathrm{Re}\,\zeta>\theta$ be arbitrary and fixed. For any $G\in\mathcal{L}_{C,\,\beta}$ it holds

$$||(\widehat{L_{\Lambda_n,\,\beta,\,\kappa}}\circ P_{\Lambda_n}-\zeta)^{-1}G-(\widehat{L_{\beta,\,\kappa}}-\zeta)^{-1}G||=$$

$$=||(\widehat{L_{\Lambda_n,\,\beta,\,\kappa}}\circ P_{\Lambda_n}-\zeta)^{-1}\left[\widehat{L_{\beta,\,\kappa}}-\widehat{L_{\Lambda_n,\,\beta,\,\kappa}}\circ P_{\Lambda_n}\right](\widehat{L_{\beta,\,\kappa}}-\zeta)^{-1}G||. \quad (16)$$

For any $G \in D(\widehat{L_{\beta,\kappa}}) = D(L_0)$

$$\left[\widehat{L_{\beta,\kappa}} - \widehat{L_{\Lambda_n,\beta,\kappa}} \circ P_{\Lambda_n}\right] G(\eta) = -|\eta| \left[1 - 1\!\!1_{\Gamma_{\Lambda_n}}(\eta)\right] G(\eta)+$$

$$+\kappa \sum_{\xi \subseteq \eta} \int_{\Lambda_n^c} G(\xi \cup x) \prod_{y \in \eta \setminus \xi} \left[e^{-\beta\phi(x-y)} - 1\right] \prod_{y' \in \xi} e^{-\beta\phi(x-y')} dx +$$

$$+\kappa \sum_{\xi \subseteq \eta} \int_{\Lambda_n} G(\xi \cup x) \prod_{y \in \eta \setminus \xi} \left[e^{-\beta\phi(x-y)} - 1\right] e^{-\beta E(x,\xi)} \times$$

$$\times \left[1 - 1\!\!1_{\Gamma_{\Lambda_n}}(\xi \cup x) 1\!\!1_{\Gamma_{\Lambda_n}}(\eta \setminus \xi)\, e^{\beta E(x,\xi_{\Lambda_n^c})}\right] dx,$$

where $\Lambda_n^c = \mathbb{R}^d \setminus \Lambda_n$.

Using fact, that for any $\xi \in \Gamma_0$ and $x \in \mathbb{R}^d$

$$1\!\!1_{\Gamma_{\Lambda_n}}(\xi \cup x)\, e^{\beta E(x,\xi_{\Lambda_n^c})} = 1\!\!1_{\Gamma_{\Lambda_n}}(\xi \cup x),$$

simple inequality

$$|1 - 1\!\!1_{\Gamma_{\Lambda_n}}(\xi) 1\!\!1_{\Gamma_{\Lambda_n}}(\eta)| \le |1 - 1\!\!1_{\Gamma_{\Lambda_n}}(\xi)| + |1 - 1\!\!1_{\Gamma_{\Lambda_n}}(\eta)|, \quad \xi,\, \eta \in \Gamma_0,$$

and estimates analogous to those which were proposed in Lemma 5.1 we obtain

$$\left|\left|\left[\widehat{L_{\beta,\kappa}} - \widehat{L_{\Lambda_n,\beta,\kappa}} \circ P_{\Lambda_n}\right] G(\eta)\right|\right| \le$$

$$\le \left(1 + \kappa \max\{1,\, C^{-1}\} e^{C(\beta)C}\right) \left|\left| \left[1 - 1\!\!1_{\Gamma_{\Lambda_n}}(\cdot)\right] \, | \cdot | G(\cdot) \right|\right| +$$

$$+\kappa \max\{1,\, C^{-1}\} e^{C(\beta)C} \left|\left| \, | \cdot_{\Lambda_n^c} | G(\cdot) \right|\right| +$$

$$+ \max\{1,\, C^{-1}\} \left|\left| \, | \cdot_{\Lambda_n} | G(\cdot) \right|\right| \int_{\Gamma_{\Lambda_n}} |1 - 1\!\!1_{\Gamma_{\Lambda_n}}(\eta)| K(0,\, \eta) C^{|\eta|} \lambda(d\eta).$$

All of the summands in the right-hand side of the last inequality definitely tends to zero, when $n \to \infty$.

Using Remark 5.5 and equality (16) we easily conclude that difference in (16) also tends to zero when $n \to \infty$. ■

6 Construction of a non-equilibrium Markov process

Fix any triple of positive constants C, κ and β which satisfies (15). Let $\widehat{U_t}(C,\beta,\kappa)$ be holomorphic semigroup generated by $\widehat{L_{\beta,\kappa}}$ and let

$$\mathcal{K}_{C,\beta} := \left\{ k : \Gamma_0 \to \mathbb{R}_+ \;|\; k(\cdot)\, C^{-|\cdot|} e^{\beta E(\cdot)} \in L^\infty(\Gamma_0, \lambda) \right\} \tag{17}$$

be the set of "so-called correlation functions". Note that $\mathcal{K}_{C,\beta}$ is a Banach space.

We introduce the following duality between quasi-observables $G \in \mathcal{L}_{C,\beta}$ and functions $k \in \mathcal{K}_{C,\beta}$

$$\langle\langle G,\, k\rangle\rangle := \langle G,\, k\rangle_{L^2(\Gamma_0,\,\lambda)}\,. \tag{18}$$

Let us mention that $G \in \mathcal{L}_{C,\beta}$ means that $G(\cdot)C^{|\cdot|}e^{-\beta E(\cdot)} \in L^1(\Gamma_0, \lambda)$. Therefore, the duality

$$\langle G,\, k\rangle_{L^2(\Gamma_0,\,\lambda)} = \int_{\Gamma_0} G(\eta)C^{|\eta|}e^{-\beta E(\eta)}k(\eta)C^{-|\eta|}e^{\beta E(\eta)})d\lambda(\eta) < \infty$$

is well-defined.

Note, also that $k(\cdot)\,C^{-|\cdot|}e^{\beta E(\cdot)} \in L^\infty(\Gamma_0, \lambda)$ means that function k satisfies the following bound

$$k(\eta) \leq const\, C^{|\eta|}e^{-\beta E(\eta)}, \tag{19}$$

which is known as *generalized Ruelle bound*, see e.g. [10].

Using duality (18) one can easily show that semigroup $\widehat{U_t}(C, \beta, \kappa)$ determines corresponding semigroup $\widehat{U_t^\star}(C, \beta, \kappa)$ on $\mathcal{K}_{C,\beta}$.

Next, we solve the following problem: suppose that $k_0 \in \mathcal{K}_{C,\beta}$ is a correlation function which means, that there exists a probability measure $\mu_0 \in \mathcal{M}^1_{fm}(\Gamma)$, locally absolutely continuous with respect to Poisson measure, whose correlation function is exactly k_0. Does evolution of k_0 with respect to the semigroup $\widehat{U_t^\star}(C, \beta, \kappa)$ preserve the property described above? Namely, will $\widehat{U_t^\star}(C, \beta, \kappa)k_0$, for any moment of time $t > 0$, be a correlation function or not?

In order to answer this problem, one can apply, for example, the theorem about characterization of correlation functions, proposed in [8]. In the model under consideration, the conditions of this theorem, which must to be checked are the following:

$$for \quad any \quad t \geq 0: \quad \left\langle\left\langle G \star G,\, \widehat{U_t^\star}(C, \beta, \kappa)k_0\right\rangle\right\rangle \geq 0, \qquad \forall\, G \in B_{\mathrm{bs}}(\Gamma_0).$$

Further explanations will be devoted to the verifying of the latter condition.

Let $\mu \in \mathcal{G}(\beta,\, z)$ and $\{\pi_{\Lambda,\,\emptyset}\}_{\Lambda\in\mathcal{B}_b(\mathbb{R}^d)}$ denote the specification with empty boundary conditions corresponding to the Gibbs measure μ. We define

$$\mathcal{E}(F,\, G) := \int_\Gamma \sum_{x\in\gamma} D_x^- F(\gamma) D_x^- G(\gamma)\pi_\Lambda(d\gamma,\, \emptyset), \quad F,\, G \in K\,CB^\Lambda_{\mathrm{bs}}(\Gamma_0).$$

Now we would like to list some facts the proofs of which are completely analogous to the proposed in [11].

Lemma 6.1 *The set $K\,CB_{\rm bs}^{\Lambda}(\Gamma_0)$ is dense in $L^2(\Gamma,\,\pi_{\Lambda,\emptyset})$ for any $\Lambda\in\mathcal{B}_b(\mathbb{R}^d)$.*

Lemma 6.2 *Let $\Lambda\in\mathcal{B}_b(\mathbb{R}^d)$ be arbitrary and fixed. Then $(\mathcal{E},\,K\,CB_{\rm bs}^{\Lambda}(\Gamma_0))$ is a well-defined bilinear form on $L^2(\Gamma,\,\pi_{\Lambda,\emptyset})$.*

Lemma 6.3 *Let $\Lambda\in\mathcal{B}_b(\mathbb{R}^d)$ be arbitrary and fixed. Suppose that conditions (I) and (P) are satisfied. Then $(L_\Lambda, K\,CB_{\rm bs}^{\Lambda}(\Gamma_0))$ is an operator associated with bilinear form $(\mathcal{E},\,K\,CB_{\rm bs}^{\Lambda}(\Gamma_0))$ in $L^2(\Gamma,\,\pi_{\Lambda,\emptyset})$, i.e.*

$$\mathcal{E}(F,\,G)=\int_\Gamma L_\Lambda F(\gamma)G(\gamma)\pi_{\Lambda,\emptyset}(d\gamma),\quad F,\,G\in K\,CB_{\rm bs}^{\Lambda}(\Gamma_0).$$

Lemma 6.4 *Let $\Lambda\in\mathcal{B}_b(\mathbb{R}^d)$ be arbitrary and fixed. Suppose that conditions (I) and (P) are satisfied and $\mu\in\mathcal{G}(z,\,\beta)$. Then there exists a self-adjoint positive Friedrichs' extension $(\widetilde{L_\Lambda},\,D(\widetilde{L_\Lambda}))$ of the operator $(L_\Lambda,\,K\,CB_{\rm bs}^{\Lambda}(\Gamma_0))$ in $L^2(\Gamma,\,\pi_{\Lambda,\emptyset})$. Moreover, $(\widetilde{L_\Lambda},\,D(\widetilde{L_\Lambda}))$ is a generator of a contraction semigroup which preserves 1 in $L^2(\Gamma,\,\pi_{\Lambda,\emptyset})$, associated with some Markov process.*

Remark 6.1 *It is well known (see e.g. [20]) that under condition of Lemma 6.4 the semigroup generated by $(\widetilde{L_\Lambda},\,D(\widetilde{L_\Lambda}))$ can be extended to the $L^1(\Gamma,\,\pi_{\Lambda,\emptyset})$. For any $\Lambda\in\mathcal{B}_b(\mathbb{R}^d)$, the extension of this semigroup in $L^1(\Gamma,\,\pi_{\Lambda,\emptyset})$ we will denote by $(\widetilde{U_t^{\Lambda}})_{t\geq 0}$. For the generator of this semigroup we will use notation $(\widetilde{L_\Lambda},\,D_1(\widetilde{L_\Lambda}))$, where $D_1(\widetilde{L_\Lambda})\supset D(\widetilde{L_\Lambda})$ is a domain of $\widetilde{L_\Lambda}$ in $L^1(\Gamma,\,\pi_{\Lambda,\emptyset})$.*

Now, we introduce one of the crucial lemma about the evolution of the "so-called correlation functions".

Lemma 6.5 *Let positive constants C, κ and β which satisfy (15) be arbitrary and fixed. The semigroup $\widehat{U_t^{\star}}(C,\beta,\kappa)$ on $\mathcal{K}_{C,\beta}$ preserves positive semi-definiteness, i.e. for any $t\geq 0$*

$$\left\langle\left\langle G\star G,\,\widehat{U_t^{\star}}(C,\beta,\kappa)k\right\rangle\right\rangle\geq 0,\qquad\forall\,G\in B_{\rm bs}(\Gamma_0)$$

iff

$$\langle\langle G\star G,\,k\rangle\rangle\geq 0,\tag{20}$$

for any $G\in B_{\rm bs}(\Gamma_0)$.

Remark 6.2 *Let $\mathcal{M}_{C,\beta}$ stands for the set of all probability measures on Γ, locally absolutely continuous with respect to Poisson measure, with locally finite moments, whose correlation functions satisfies bound (19). As it was pointed out at the beginning of this section, the condition (20) on function $k\in\mathcal{K}_{C,\beta}$, insures an existence of a unique measure $\mu^k\in\mathcal{M}_{C,\beta}$ whose correlation function is k, see [8].*

Proof of Lemma 6.5. Under assumptions of the lemma we have to show that for any $t \geq 0$

$$\left\langle\left\langle \widehat{U_t}(C,\beta,\kappa)(G\star G),\, k\right\rangle\right\rangle \geq 0, \qquad \forall\, G \in B_{\mathrm{bs}}(\Gamma_0). \tag{21}$$

But $G\star G \in B_{\mathrm{bs}}(\Gamma_0)$ for any $G \in B_{\mathrm{bs}}(\Gamma_0)$. Therefore, due to Theorem 5.3 it is enough to show that for any $t \geq 0$ and any $G \in B_{\mathrm{bs}}(\Gamma_0)$ there exists $\Lambda' \in \mathcal{B}_b(\mathbb{R}^d)$ such that for all $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$, $\Lambda \supset \Lambda'$

$$\left\langle\left\langle \widehat{U_t^\Lambda}(C,\beta,\kappa) \circ P_\Lambda(G\star G),\, k\right\rangle\right\rangle \geq 0. \tag{22}$$

Let $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$ be arbitrary and fixed. We set

$$U_t^\Lambda := K\widehat{U_t^\Lambda}(C,\beta,\kappa)K^{-1}, \quad t \geq 0.$$

$(U_t^\Lambda)_{t\geq 0}$ is a semigroup on

$$(\mathcal{L}_1^\Lambda := K\mathcal{L}_{C,\,\beta}^\Lambda,\, ||\cdot||_1 := ||K^{-1}\cdot||_{\mathcal{L}_{C,\,\beta}})$$

which is the Banach space. Moreover, it is not difficult to show that a generator of this semigroup coincide with $(L_\Lambda,\, K\, D(\widehat{L_\Lambda}))$.

Proposition 6.1 *For any* $F \in \mathcal{L}_1^\Lambda \subset L^1(\Gamma,\, \pi_{\Lambda,\emptyset})$,

$$U_t^\Lambda F = \widetilde{U_t^\Lambda} F, \quad t \geq 0 \quad in \quad L^1(\Gamma,\, \pi_{\Lambda,\emptyset}).$$

Proof. The fact that $(L_\Lambda,\, K\, D(\widehat{L_\Lambda}))$ is a generator of $(U_t^\Lambda)_{t\geq 0}$ in $(\mathcal{L}_1^\Lambda,\, ||\cdot||_1)$ means the following (see e.g. [6])

$$\left|\left| U_t^\Lambda F - \left(\frac{t}{n}L_\Lambda - 1\!\!1\right)^{-n} F\right|\right|_1 \to 0, \quad n\to\infty, \quad \text{for all } F \in \mathcal{L}_1^\Lambda.$$

Because $||\cdot||_1 \geq ||\cdot||$, the latter fact implies

$$\left|\left| U_t^\Lambda F - \left(\frac{t}{n}L_\Lambda - 1\!\!1\right)^{-n} F\right|\right| \to 0, \quad n\to\infty, \quad \text{for all } F \in \mathcal{L}_1^\Lambda. \tag{23}$$

Analogously, the fact that $(\widetilde{L_\Lambda},\, D_1(\widetilde{L_\Lambda}))$ is a generator of $(\widetilde{U_t^\Lambda})_{t\geq 0}$ gives us

$$\left|\left| \widetilde{U_t^\Lambda} F - \left(\frac{t}{n}\widetilde{L_\Lambda} - 1\!\!1\right)^{-n} F\right|\right| \to 0, \quad n\to\infty, \quad \text{for all } F \in \mathcal{L}_1^\Lambda. \tag{24}$$

As was shown before, there exists $\epsilon > 0$ such that for any real $\zeta > \epsilon$ and any $F \in \mathcal{L}_1^\Lambda$

$$\left(\widetilde{L_\Lambda} - \zeta 1\!\!1\right)^{-1} F - (L_\Lambda - \zeta 1\!\!1)^{-1} F = \left(\widetilde{L_\Lambda} - \zeta 1\!\!1\right)^{-1} \left[L_\Lambda - \widetilde{L_\Lambda}\right] (L_\Lambda - \zeta 1\!\!1)^{-1} F.$$

The function $F_\zeta := (L_\Lambda - \zeta 1\!\!1)^{-1} F \in K\, D(\widehat{L_\Lambda})$. Hence, $\left[L_\Lambda - \widetilde{L_\Lambda}\right] F_\zeta = 0$. The latter fact means that

$$\left\|\widetilde{U_t^\Lambda} F - \left(\frac{t}{n}\widetilde{L_\Lambda} - 1\!\!1\right)^{-n} F\right\| =$$

$$= \left\|\widetilde{U_t^\Lambda} F - \left(\frac{t}{n} L_\Lambda - 1\!\!1\right)^{-n} F\right\| \to 0, \quad n \to \infty, \quad \text{for all } \; F \in \mathcal{L}_1^\Lambda. \tag{25}$$

The convergence (23) and (24) imply the assertion of the proposition. ■

Corollary 6.1 *Lemma 6.4 implies that for any moment of time* $t \geq 0$

$$U_t^\Lambda F \geq 0, \quad \textit{for all} \;\; F \geq 0 \;\; \textit{in} \;\; L^1(\Gamma,\, \pi_{\Lambda,\emptyset}). \tag{26}$$

Let $t \geq 0$ and $G \in B_{\mathrm{bs}}(\Gamma_0)$ be arbitrary and fixed. Suppose that $N' \in \mathbb{N}$ and $\Lambda' \in \mathcal{B}_b(\mathbb{R}^d)$ are such that

$$G \star G \restriction_{\Gamma_0 \setminus \sqcup_{n=0}^{N'} \Gamma_{\Lambda'}^{(n)}} = 0.$$

Then, $K(G \star G) = |KG|^2 \in \mathcal{L}_1^\Lambda$ for all $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$, $\Lambda \supset \Lambda'$. Moreover, $P_\Lambda |KG|^2 = |KG|^2$.

Hence, the left-hand side of (22) for any $\Lambda \in \mathcal{B}_b(\mathbb{R}^d)$, $\Lambda \supset \Lambda'$ is equal to the following expression

$$\left\langle\!\left\langle \widehat{U_t^\Lambda}(C, \beta, \kappa) \circ P_\Lambda(G \star G),\, k \right\rangle\!\right\rangle = \int_\Gamma K \widehat{U_t^\Lambda}(G \star G)(\gamma) \mu^k(d\gamma) =$$

$$= \int_\Gamma U_t^\Lambda K(G \star G)(\gamma) \mu^k(d\gamma) = \int_{\Gamma_\Lambda} U_t^\Lambda |KG|^2(\gamma) \mu_\Lambda^k(d\gamma),$$

where μ_Λ^k is a projection of μ^k on Γ_Λ. Let us mention that measure μ^k is locally absolutely continuous with respect to Poisson measure π. Therefore,

$$\left\langle\!\left\langle \widehat{U_t^\Lambda}(C, \beta, \kappa) \circ P_\Lambda(G \star G),\, k \right\rangle\!\right\rangle = \int_{\Gamma_\Lambda} U_t^\Lambda |KG|^2(\gamma) \frac{d\mu_\Lambda^k}{d\pi_\Lambda}(\gamma) \pi_\Lambda(d\eta).$$

Corollary 6.1 implies that there exist set $S \subset \Gamma$, $\pi_{\Lambda,\emptyset}(S) = 0$ such that for all $\gamma \in \Gamma \setminus S$:

$$U_t^\Lambda |KG|^2(\gamma) \geq 0.$$

But $\pi_{\Lambda,\emptyset}$ is absolutely continuous with respect to π_Λ. Furthermore, the corresponding Radon-Nikodim derivative is positive almost surely with respect to π_Λ. Hence, $\pi_\Lambda(S_\Lambda) = 0$, where S_Λ is a projection of the set S to Γ_Λ, and

$$\left\langle\left\langle \widehat{U_t^\Lambda}(C,\beta,\kappa) \circ P_\Lambda(G \star G),\, k\right\rangle\right\rangle = \int_{\Gamma_\Lambda \setminus S_\Lambda} U_t^\Lambda |KG|^2(\gamma) \frac{d\mu_\Lambda^k}{d\pi_\Lambda}(\gamma)\pi_\Lambda(d\eta) \geq 0.$$

The latter proof the assertion of Lemma 6.5. ■

The result obtained in Lemma 6.5 and fact about characterization of correlation functions from [8] imply the following corollary.

Corollary 6.2 *Let positive constants C, κ and β which satisfy (15) be arbitrary and fixed. Let $k \in \mathcal{K}_{C,\beta}$ be such that $\langle\langle G \star G,\, k\rangle\rangle \geq 0$, for any $G \in B_{\mathrm{bs}}(\Gamma_0)$. Then for any $t \geq 0$ there exists unique measure $\mu_t \in \mathcal{M}_{C,\beta}$ whose correlation function is $\widehat{U_t^\star}(C,\beta,\kappa)k$.*

Let us denote in Corollary 6.2 the evolution of the measure μ in time by $U_t^\star(C,\beta,\kappa)\mu := \mu_t$. One can easily show that $(U_t^\star(C,\beta,\kappa))_{t\geq 0}$ is a semigroup on $\mathcal{M}_{C,\beta}$. This leads us directly to the construction of the non-equilibrium Markov process (or rather Markov function) on Γ.

Theorem 6.1 *Suppose that conditions (I) and (P) are satisfied. For any triple of positive constants C, κ and β which satisfy (15) and any $\mu \in \mathcal{M}_{C,\beta}$ there exists Markov process $X_t^\mu \in \Gamma$ with initial distribution μ associated with generator $L_{\beta,\kappa}$.*

Proof. Let $n \in \mathbb{N}$, functions $0 \leq F_0, F_1, \ldots, F_n \in L^\infty(\Gamma)$ and moments of time $0 \leq t_0 \leq t_1 \leq \ldots \leq t_n$ be any and fixed. Then there exists a process, defined on some probability space $(\Omega, \mathcal{F}, P)$, the finite-dimensional distribution of which is given by the following formula:

$$\int_\Omega F_0(X_{t_0}^\mu)\ldots F_n(X_{t_n}^\mu)dP := \int_\Gamma dF_n \ldots U_{t_1-t_0}^\star(C,\kappa,\eta)(F_0\mu)$$

Eventually, we have constructed the non-equilibrium Markov process. ■

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