Volatile decision dynamics: experiments, stochastic description, intermittency control and traffic optimization

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Abstract. The coordinated and efficient distribution of limited resources by individual decisions is a fundamental, unsolved problem. When individuals compete for road capacities, time, space, money, goods, etc, they normally make decisions based on aggregate rather than complete information, such as TV news or stock market indices. In related experiments, we have observed a volatile decision dynamics and far-from-optimal payoff distributions. We have also identified methods of information presentation that can considerably improve the overall performance of the system. In order to determine optimal strategies of decision guidance by means of user-specific recommendations, a stochastic behavioural description is developed. These strategies manage to increase the adaptibility to changing conditions and to reduce the deviation from the time-dependent user equilibrium, thereby enhancing the average and individual payoffs. Hence, our guidance strategies can increase the performance of all users by reducing overreaction and stabilizing the decision dynamics. These results are highly significant for predicting decision behaviour, for reaching optimal behavioural distributions by decision support systems and for information service providers. One of the promising fields of application is traffic optimization.

1. Introduction

Optimal route guidance strategies in overloaded traffic networks, for example, require reliable traffic forecasts (see figure 1). These are extremely challenging for two reasons. First of all, traffic dynamics is very complex. However, after more than 50 years of traffic research, physicists have recently gained at least a semi-quantitative understanding of it based on the concept of self-driven, nonlinearly interacting many-particle systems [2]. The second and more serious problem is the invalidation of forecasts by the driver reactions to route choice recommendations. Nevertheless, some keen scientists hope to solve this long-standing problem by means of an iteration scheme [3]–[11]: if the driver reaction was known from experiments [12]–[22], the resulting traffic situation could be calculated, yielding improved route choice recommendations, etc. Given this iteration scheme converges, it would facilitate optimal recommendations and reliable traffic forecasts anticipating the driver reactions. Based on empirically determined transition and compliance probabilities, we will develop a new procedure in the following, which would even allow us to reach the optimal traffic distribution in a single step and in harmony with the forecast.
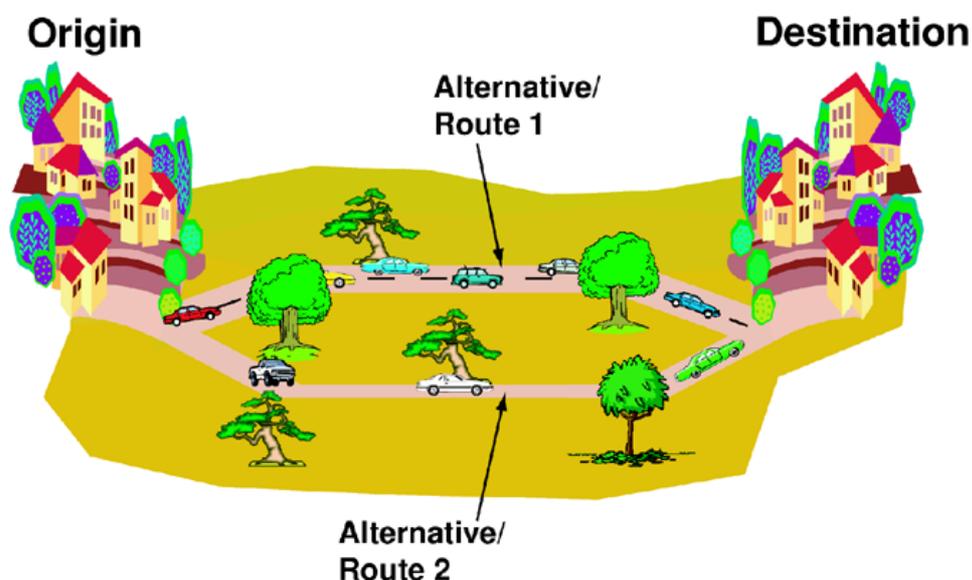

Figure 1. Schematic illustration of a day-to-day route choice scenario. Each day, the drivers have to decide between two alternative routes, 1 and 2. Note that, due to the different number of lanes, route 1 has a higher capacity than route 2. The latter is, therefore, used by less cars.

The solution of this difficult and practically relevant problem requires several concepts and methods from physics. First of all, we will identify the essential role and reason for *intermittency* in scenarios with repeated decisions (see section 3). Second, we will derive a *nonlinear feedback mechanism* for *intermittency control* (see section 4). In addition, we will develop a stochastic description of the decision behaviour (see section 5) and evaluate the corresponding transition and compliance probabilities including their time dependence (see section 4).

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2. Experimental set-up and previous results

To determine the route choice behaviour, Schreckenberg *et al* [19] have recently carried out a decision experiment [1] (see figure 2). N test persons had to repeatedly decide between two alternatives 1 and 2 (the routes) and should try to maximize their resulting payoffs (describing something like the speeds or inverse travel times). To reflect the competition for a limited resource (the road capacity) the received payoffs

$$P_1(n_1) = P_1^0 - P_1^1 n_1 \qquad \text{and} \qquad P_2(n_2) = P_2^0 - P_2^1 n_2 \qquad (1)$$

decreased with the numbers of test persons n_1 and $n_2 = N - n_1$ deciding for alternatives 1 and 2, respectively. The *user equilibrium* corresponding to equal payoffs for both alternative decisions is found for a fraction

$$f_1^{\text{eq}} = \frac{n_1}{N} = \frac{P_2^1}{P_1^1 + P_2^1} + \frac{1}{N} \frac{P_1^0 - P_2^0}{P_1^1 + P_2^1} \qquad (2)$$
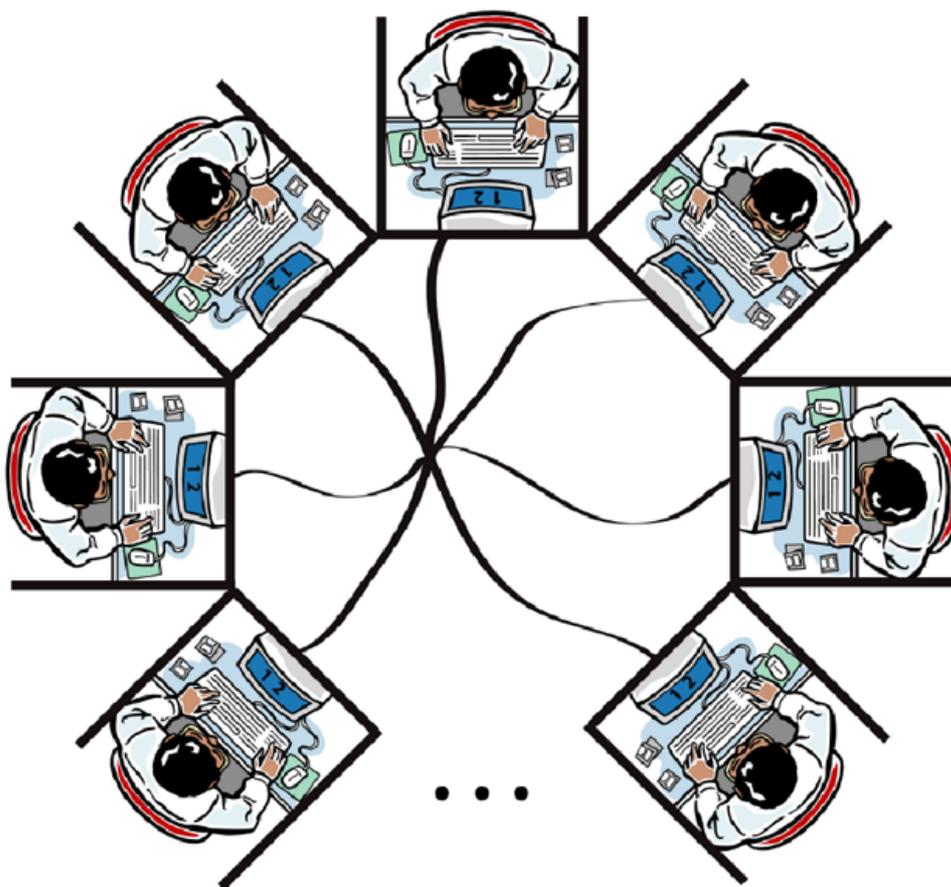

Figure 2. Schematic illustration of the decision experiment. Several test persons have to take decisions based on the aggregate information their computer displays. The computers are connected and can, therefore, exchange information. However, a direct communication among players is suppressed.

of persons choosing alternative 1. The *system optimum* corresponds to the maximum of the total payoff $n_1 P_1(n_1) + n_2 P_2(n_2)$, which lies by an amount

$$\frac{1}{2N} \frac{P_1^0 - P_2^0}{P_1^1 + P_2^1} \tag{3}$$

below the user optimum. Therefore, only experiments with a few players allow one to find out whether the test persons adapt to the user or the system optimum. Small groups are also more suitable for the experimental investigation of the fluctuations in the system and of the long-term adaptation behaviour. Schreckenberg *et al* found that, on average, the test groups adapted relatively well to the user equilibrium. However, although it appears reasonable to stick to the same decision once the equilibrium is reached, the standard deviation stayed at a finite level. This was not only observed in *'treatment' 1*, where all players knew only their own (previously experienced) payoff, but also in *treatment 2*, where the payoffs $P_1(n_1)$ and $P_2(n_2)$ for both, 1- *and* 2-decisions, were transmitted to all players (analogous to radio news). Nevertheless, treatment 2 could decrease the changing rate and increase the average payoffs. For details regarding the statistical analysis see [19].

To explain the mysterious persistence in the changing behaviour and explore possibilities to suppress it, we have repeated these experiments with more iterations and tested additional treatments. In the beginning, all treatments were consecutively applied to the same players in order to determine the response to different kinds of information (see figure 3). Afterwards, single treatments and variants of them were repeatedly tested with different players to check our conclusions. Apart from this, we have generalized the experimental set-up in the sense that it was no longer restricted to route choice decisions: the test persons did not have any idea of the payoff functions in the beginning, but had to develop their own hypothesis about them. In particular, the players did not know that the payoff decreased with the number of persons deciding for the same alternative.

In *treatment 3*, every test person was informed about the own payoff $P_1(n_1)$ (or $P_2(n_2)$) and the *potential payoff*

$$P_2(N - n_1 + \epsilon N) = P_2(n_2) - \epsilon N P_2^1 \tag{4}$$

(or $P_1(N - n_2 + \epsilon N) = P_1(n_1) - \epsilon N P_1^1$) he or she would have obtained, if a fraction ϵ of persons had additionally chosen the other alternative (here $\epsilon = 1/N = 1/9$). Treatments 4 and 5 were variants of treatment 3, but some payoff parameters were changed in time to simulate varying environmental conditions. In *treatment 5* each player additionally received an individual recommendation which provided an alternative choice.

The higher changing rate in treatment 1 compared to treatment 2 can be understood as the effect of an exploration rate ν_1 required to find out which alternative performs better. It is also plausible that treatment 3 could further reduce the changing rate: in the user equilibrium with $P_1(n_1) = P_2(n_2)$, every player knew that he or she would *not* get *the same*, but a *reduced* payoff, if he or she would change the decision. That explains why the new treatment 3 could reach a great adaptation performance, reflected by a very low standard deviation and almost optimal average payoffs. The behavioural changes induced by the treatments were not only observed on average, but for every single individual (see figure 4). Moreover, even the smallest individual cumulative payoff exceeded the highest one in treatment 1. Therefore, treatment 3's method of information presentation is far superior to the ones used today.
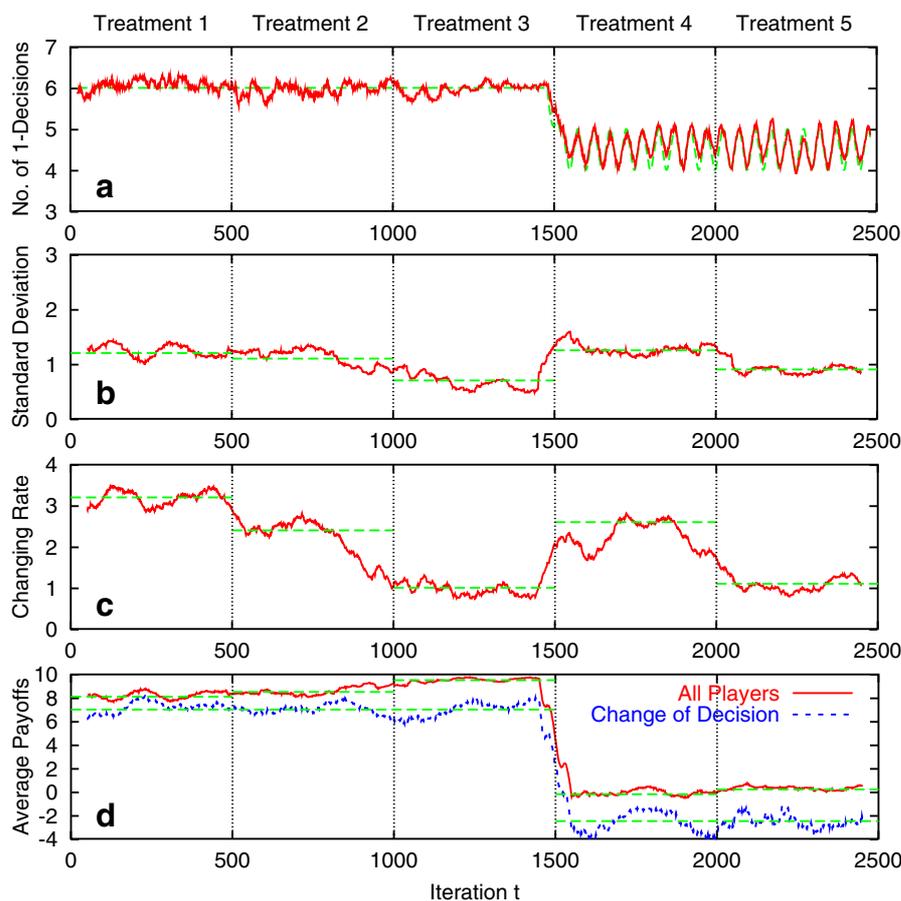
Figure 3. Overview of treatments 1–5 (with $N = 9$ and payoff parameters $P_2^0 = 28$, $P_1^1 = 4$, $P_2^1 = 6$ and $P_1^0 = 34$ for $0 \leq t \leq 1500$, but a zig-zag-like variation between $P_1^0 = 44$ and $P_1^0 = -6$ with a period of 50 for $1501 \leq t \leq 2500$): (a) the average number of decisions for alternative 1 (solid curve) compared to the user equilibrium (dashed curve), (b) the standard deviation of the number of 1-decisions from the user equilibrium, (c) the number of decision changes from one iteration to the next one, (d) the average payoff per iteration for players who have changed their decision and for all players. The latter increased with a reduction in the changing rate, but normally stayed below the payoff in the user equilibrium (which is 1 on average in treatments 4 and 5, otherwise 10). The displayed moving time averages ((a) over 40 iterations, (b)–(d) over 100 iterations) illustrate the systematic response to changes in the treatment every 500 iterations. The dashed lines in (b)–(d) show estimates of the stationary values after the transient period (to guide the eyes), while time periods around the dotted lines are not significant. Compared to treatment 1, treatment 3 managed to reduce the changing rate and to increase the average payoffs (three times more than treatment 2 did). These changes were systematic for *all* players (see figure 4). In treatment 4, the changing rate and the standard deviation increased, since the user equilibrium changed in time. The user-specific recommendations in treatment 5 could almost fully compensate for this. The above conclusions are also supported by additional experiments with single treatments.
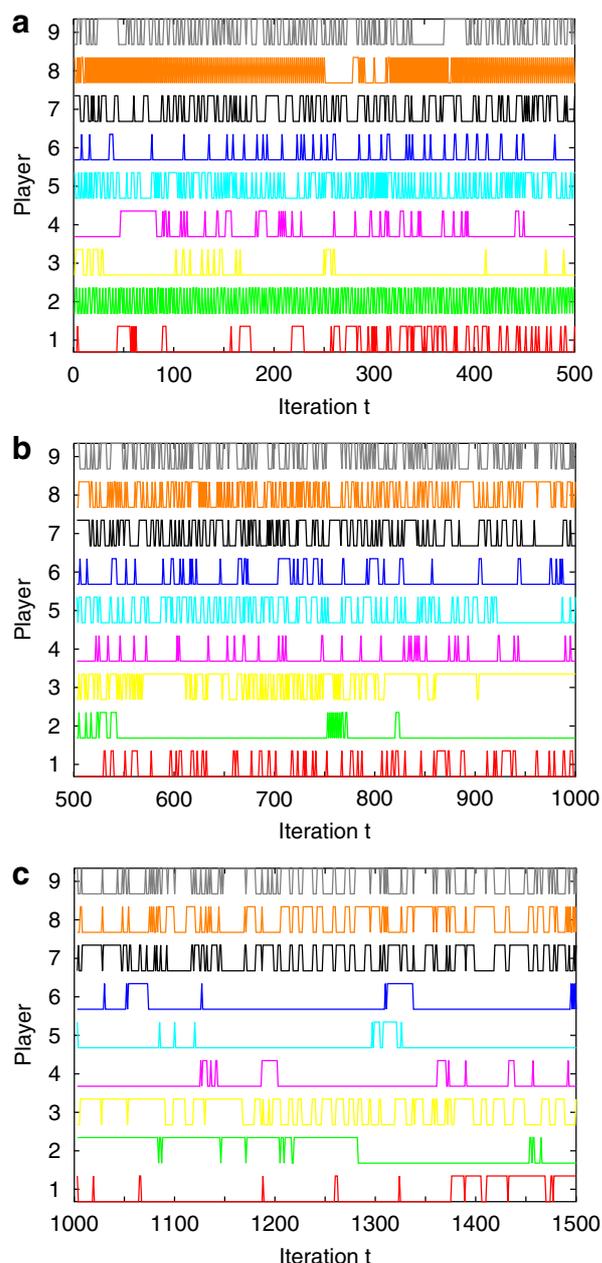
Figure 4. Comparison of the individual decision behaviours under (a) treatment 1, (b) treatment 2 and (c) treatment 3. The upper values correspond to a decision for alternative 2, the lower ones for alternative 1. Note that some test persons showed similar behaviours (either more or less the same or almost opposite ones), although they could not talk to each other. This shows that there are some typical strategies how to react to specific information. The group has, in fact, to develop complementary strategies in order to reach a good adaptation performance. Identical strategies would perform poorly (as in the minority game [23]–[26]). Despite the mentioned complementary behaviour, there is a characteristic reaction to changes in the treatment. For example, compared to treatment 2 all players reduce their changing rate in treatment 3.

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3. Explaining the volatile decision dynamics

In this section, we will investigate why players changed their decision in the user equilibrium at all. The reason for the pertaining changing behaviour can be revealed by a more detailed analysis of the individual decisions in treatment 3. Figure 5 shows some kind of intermittent behaviour, i.e. quiescent periods without changes, followed by turbulent periods with many changes. This is reminiscent of volatility clustering in stock market indices [27]–[29], where individuals also react to aggregate information reflecting all decisions (the trading transactions). Single players seem to change their decision to reach above-average payoffs. In fact, although the cumulative individual payoff is anticorrelated with the average changing rate, some players receive higher payoffs with larger changing rates than others. They profit from the overreaction in the system. Once the system is out of equilibrium, all players respond in one way or another. Typically, there are too many decision changes (see figures 5 and 6). The corresponding overcompensation, which had also been predicted by computer simulations [5, 8]–[10, 17, 30], gives rise to 'turbulent' periods.

We should, however, note that the calm periods without decision changes tend to become longer over the course of time. That is, after a very long time period the individuals learn not to change their behaviour when the user equilibrium is reached. This is not only found in figure 5, but is also visible in figure 3(c) after about 800 iterations. In larger systems (with more participants) this transient period would take even longer, so that this stabilization effect cannot be observed in experiments with less iterations or more test persons.

Finally, we should stress that other interpretations of the rather persistent decision changes have been ruled out, for example, an unstable user equilibrium or a competition between the user optimum and the system optimum. High changing rates also occur if the user and system equilibrium agree, and if the payoff functions $P_1(n_1)$ and $P_2(n_2)$ are the same (i.e. $f_1^{eq} = 1/2$).

4. Decision and intermittency control by nonlinear feedback based on guidance strategies

To avoid overreaction, in treatment 5 we have recommended a number $f_1^{eq}(t+1)N - n_1(t)$ of players to change their decision and the others to remain unchanged. These user-specific recommendations helped the players to reach the smallest overreaction of all treatments (see figure 6) and a very low standard deviation, although the payoffs were changing in time (see figure 7). Treatment 4 shows how the group performance was affected by the time-dependent user equilibrium: even without recommendations, the group managed to adapt to the changing conditions surprisingly well, but the standard deviation and changing rate were approximately as high as in treatment 2 (see figure 3). This adaptability (the collective 'group intelligence') is based on complementary responses (direct and contrary ones [19], 'movers' and 'stayers', cf figure 4). That is, if some players do not react to the changing conditions, others will take the chance to earn additional payoff. This experimentally supports the behaviour assumed in the theory of efficient markets, but here the efficiency is limited by overreaction.

In most experiments, we found a constant and high compliance $C_S(t) \approx 0.92$ with recommendations to stay, but the compliance $C_M(t)$ with recommendations to change (to 'move') [15, 16, 31, 32] turned out to vary in time. It decreased with the reliability of the recommendations (see figure 8(a)), which again dropped with the compliance.
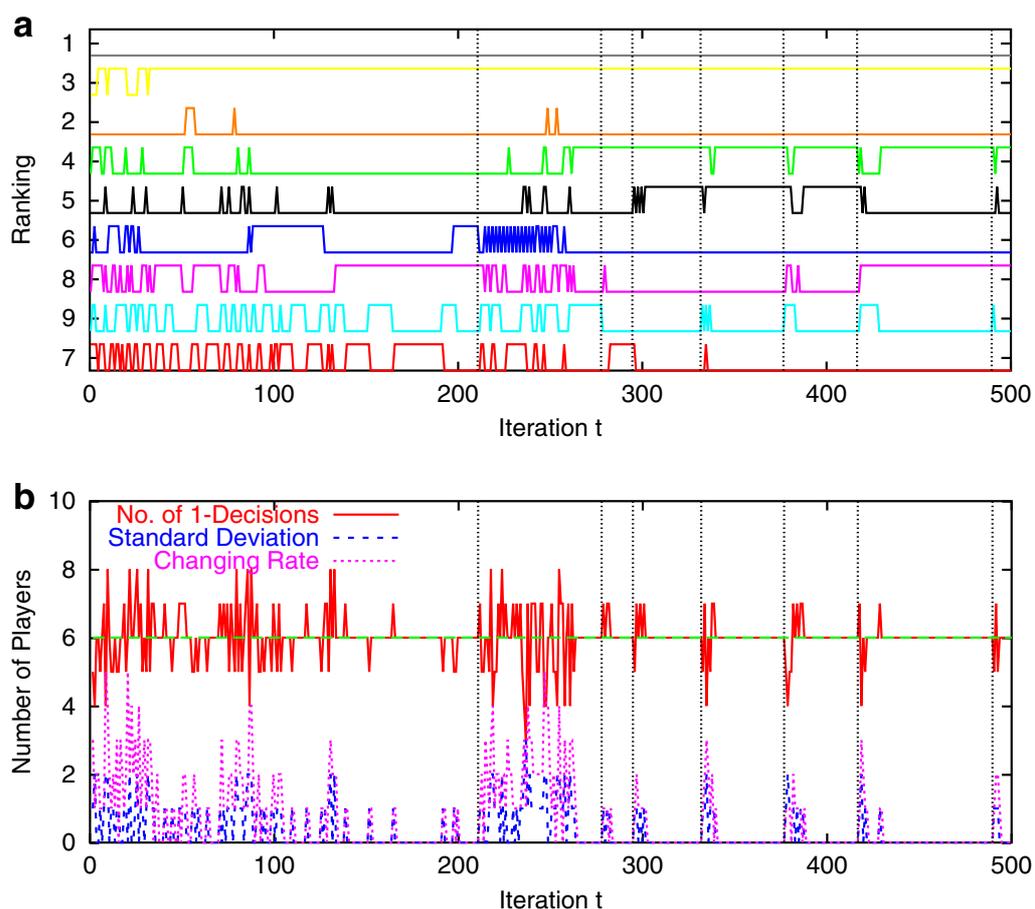
Figure 5. Illustration of typical results for treatment 3 (which here was the only treatment applied to the test persons, in contrast to figure 3). (a) The decisions of all nine players. Players are displayed from the top to the bottom in the order of increasing changing rate. Although the ranking of the cumulative payoff and the changing rate are anticorrelated, the relation is not monotonic. Note that turbulent or volatile periods characterized by many decision changes are usually triggered by individual changes after quiescent periods (dotted lines). (b) The changing rate is mostly larger than the (standard) deviation from the user equilibrium $n_1 = f_1^{\mathrm{eq}} N = 6$, indicating an overreaction in the system.

Based on this knowledge, we have developed a model which deals with how the competition for limited resources (such as road capacity) could be *optimally* guided by means of information services. Let us assume we had $n_1(t)$ 1-decisions at time t, but the optimal number of 1-decisions at time $t + 1$ is calculated to be $f_1^{\mathrm{eq}}(t + 1)N \geq n_1(t)$. Our aim is to balance the deviation $f_1^{\mathrm{eq}}(t + 1)N - n_1(t) \geq 0$ by the expected net number

$$\langle \Delta n_1(t+1) \rangle = \langle n_1(t+1) - n_1(t) \rangle = \langle n_1(t+1) \rangle - n_1(t) \tag{5}$$

of transitions from decision 2 to 1, i.e. $f_1^{\mathrm{eq}}(t + 1)N - n_1(t) = \langle \Delta n_1(t + 1) \rangle$. In the case $f_1^{\mathrm{eq}}(t + 1)N - n_1(t) < 0$, indices 1 and 2 have to be interchanged.
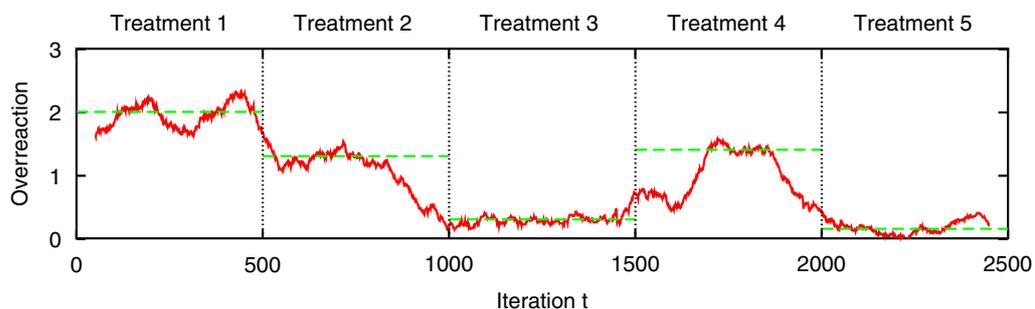
Figure 6. Measured overreaction, i.e. the difference between the actual number of decision changes (the changing rate) and the required number (the standard deviation). The overreaction can be significantly influenced by the treatment, i.e. the method of information presentation. The minimum overreaction was reached by treatment 5, i.e. user-specific recommendations.
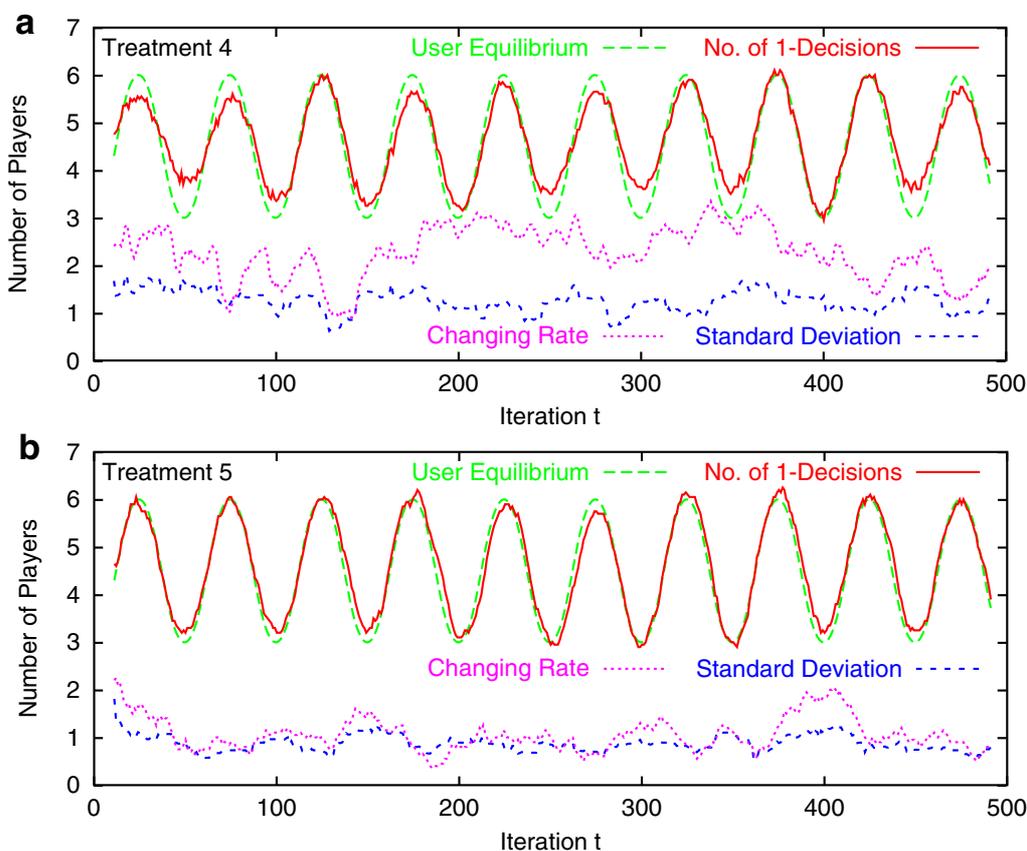

Figure 7. Representative examples for (a) treatment 4 and (b) treatment 5. The displayed curves are moving time-averages over 20 iterations. Compared to treatment 4, the user-specific recommendations in treatment 5 (assuming $C_M = C_S = 1$, $R_1 = 0$, $R_2 = \max([f_1^{\text{eq}}(t+1)N - n_1(t) + B(t+1)]/n_2(t), 1)$, $I_1 = I_2 = 1$) could increase the group adaptability to the user equilibrium a lot, even if they had a systematic or random bias B (see figure 8(a)). The standard deviation was reduced considerably and the changing rate even more.
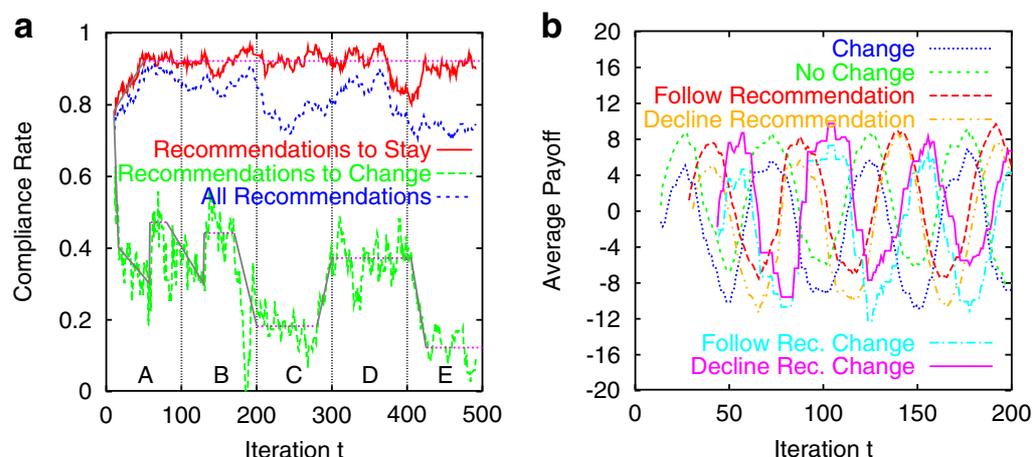
Figure 8. (a) In treatment 5, the compliance to recommendations to change dropped considerably below the compliance to recommendations to stay. The compliance to changing recommendations was very sensitive to the degree of their reliability, i.e. participants followed recommendations just as much as they helped them to reach the user equilibrium (so that the bias B did not affect the small deviation from it, see figure 7(b). While during the time interval A, the recommendations would have been perfect, if all players had followed them, in the time interval B the user equilibrium was overestimated by $B = +1$, in C it was underestimated by $B = -2$, in D it was randomly over- or underestimated by $B = \pm 1$, and in E by $B = \pm 2$. Obviously, a random error is more serious than a systematic one of the same amplitude. Dotted non-vertical lines illustrate the estimated compliance levels during and following the transient periods (horizontal dotted lines). (b) The average payoffs varied largely with the decision behaviour. Players who changed their decision got significantly lower payoffs on average than those who kept their previous decision. Even recommendations could not overcome this difference: it stayed profitable not to change, although it was generally better to follow recommendations than to refuse them. For illustrative reasons, the third and fourth lines were shifted by 15 iterations, while the fifth and sixth lines were shifted by 30 iterations.

Let us assume we give recommendations to fractions $I_1(t)$ and $I_2(t)$ of players who had chosen decisions 1 and 2, respectively. The fraction of changing recommendations to previous 1-choosers shall be denoted by $R_1(t)$, and for previous 2-choosers by $R_2(t)$. Correspondingly, fractions of $[1 - R_1(t)]$ and $[1 - R_2(t)]$ receive a recommendation to stick to the previous decision. Moreover, $[1 - C_M(t)]$ is the *refusal probability* of recommendations to change, while $[1 - C_S(t)]$ is the refusal probability of recommendations to stay. Finally, we denote the spontaneous transition probability from decision 1 to 2 by $p_a(2|1, n_1; t)$ and the inverse transition probability by $p_a(1|2, n_1; t)$, in case a player does not receive any recommendation. This happens with probabilities $[1 - I_1(t)]$ and $[1 - I_2(t)]$, respectively. Both transition probabilities $p_a(2|1, n_1; t)$ and $p_a(1|2, n_1; t)$ are functions of the number $n_1(t) = N - n_2(t)$ of previous 1-decisions. The index a allows us to reflect different strategies or characters of players. The fraction of players

pursuing strategy a is then denoted by $F_a(t)$. Applying methods summarized in [33], the expected change $\langle \Delta n_1(t+1) \rangle$ of n_1 is given by the balance equation

$$
\begin{aligned}
\langle \Delta n_1(t+1) \rangle = &\sum_a p_a(1|2, n_1; t) F_a(t)[1 - I_2(t)]n_2(t) \\
&- \sum_a p_a(2|1, n_1; t) F_a(t)[1 - I_1(t)]n_1(t) \\
&+ \sum_a \{C_M^a(t)R_2(t) + [1 - C_S^a(t)][1 - R_2(t)]\} F_a(t)I_2(t)n_2(t) \\
&- \sum_a \{C_M^a(t)R_1(t) + [1 - C_S^a(t)][1 - R_1(t)]\} F_a(t)I_1(t)n_1(t).
\end{aligned}
\tag{6}
$$

Together with the requirement

$$
\langle \Delta n_1(t+1) \rangle = f_1^{\text{eq}}(t+1)N - n_1(t),
\tag{7}
$$

this equation defines, with respect to the number n_1 of previous 1-decisions, a *nonlinear feedback* or *control strategy*.

Note that, for equation (6), it was not necessary to distinguish different characters a. We have, therefore, evaluated the overall transition probabilities

$$
p(1|2, n_1; t) = \sum_a p_a(1|2, n_1; t) F_a(t) \qquad \text{and} \qquad p(2|1, n_1; t) = \sum_a p_a(2|1, n_1; t) F_a(t).
\tag{8}
$$

According to classical decision theories [33]–[36], we would expect that the transition probabilities $p_a(1|2, n_1; t)$ and $p(1|2, n_1; t)$ should be monotonically increasing functions of the payoff $P_1(n_1(t))$, the payoff difference $P_1(n_1(t)) - P_2(N - n_1(t))$, the potential payoff $P_1(n_1(t) + \epsilon N)$, or the potential payoff gain $P_1(n_1(t) + \epsilon N) - P_2(N - n_1(t))$. All these quantities vary linearly with n_1, so that $p(1|2, n_1; t)$ should be a monotonic function of $n_1(t)$. A similar thing should apply to $p(2|1, n_1; t)$. Instead, the experimental data point to transition probabilities with a *minimum* at the user equilibrium (see figure 9(a)). That is, the players stick to a certain alternative for a longer time, when the system is close to the user equilibrium. This is a result of learning [37] (see also [38]–[42]). In fact, we find a gradual change of the transition probabilities in time (see figure 9(b)). The corresponding 'learning curves' reflect the players' adaptation to the user equilibrium.

After the experimental determination of the transition probabilities $p(2|1, n_1; t)$, $p(1|2, n_1; t)$ and specification of the overall compliance probabilities

$$
C_M(t) = \sum_a C_M^a(t) F_a(t), \qquad C_S(t) = \sum_a C_S^a(t) F_a(t),
\tag{9}
$$

we can guide the decision behaviour in the system via the levels $I_i(t)$ of information dissemination and the fractions $R_i(t)$ of recommendations to change ($i \in \{1, 2\}$). These four degrees of freedom allow us to apply a variety of guidance strategies depending on the respective information medium. For example, a guidance by radio news is limited by the fact that $I_1(t) = I_2(t)$ is given by the average percentage of radio users. Therefore, equations (6) and (7) cannot always be solved by variation of the fractions of changing recommendations $R_i(t)$. User-specific services have much higher guidance potentials and could, for example, be transmitted via SMS. Among the different guidance strategies fulfilling equations (6) and (7), the one with the minimal statistical variance will be the best. However, it would already improve the present situation to inform everyone about the *fractions $R_i(t)$* of participants who should change their decision, as users can learn to respond with varying probabilities (see figure 9).
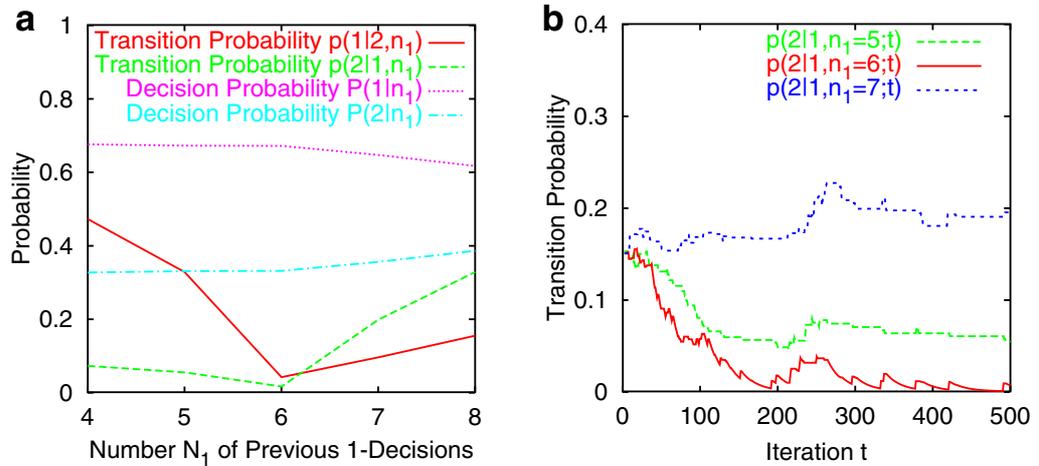
Figure 9. Illustration of decision distributions $P(i|n_1)$ and transition probabilities $p(i'|i, n_1; t)$ measured in treatment 3. (a) The probability $P(1|n_1)$ to choose alternative 1 was approximately 2/3, independent of the number n_1 of players who had previously chosen alternative 1. The probability $P(2|n_1)$ to choose alternative 2, given that n_1 players had chosen alternative 1, was always about 1/3. In contrast, the transition probability $p(1|2, n_1)$ describing decision changes from alternative 2 to 1 did depend on the number n_1 of players who had chosen decision 1. The same was true for the inverse transition probability $p(2|1, n_1)$ from decision 1 to 2. Remarkably enough, these transition probabilities are not monotonically increasing with the payoff or the expected payoff gain, as they do not monotonically increase with n_1. Instead, the probability of changing the decision shows a minimum at the user equilibrium $n_1 = f_1^{\text{eq}}N = 6$. (b) The reason for the different transition probabilities is an adaptation process in which the participants learn to take fewer changing decisions, when the user equilibrium is reached or close by, but more, when the user equilibrium is far away. (The curves were exponentially smoothed with $\alpha = 0.05$.)

The outlined guidance strategy could, of course, also be applied to reach the system optimum rather than the user optimum. The values of $\Delta n_1(t+1)$ would just be different. Note, however, that the users would soon recognize that this guidance is not suitable to reach the user optimum. Consequently, the compliance probabilities $C_j(t)$ with $j \in \{M, S\}$ would gradually decrease, which would affect the potentials and reliability of the guidance system. This problem can only be solved by a suitable modification of the payoff functions, adapting the user optimum to the system optimum.

In practical applications, we would determine the time-dependent compliance probabilities $C_j(t)$ (and the transition probabilities) on-line with an exponential smoothing procedure according to

$$C_j(t+1) = \alpha C_j'(t) + (1-\alpha)C_j(t) \qquad \text{with } \alpha \approx 0.1, \tag{10}$$

where $C_j'(t)$ is the percentage of participants who have followed their recommendation at time t. As the average payoff for decision changes is normally lower than for staying with the previous decision (see figures 8(b) and 3(d)), a high compliance probability C_M is hard to achieve.

That is, individuals who follow recommendations to change normally pay for reaching the user equilibrium (because of the overreaction in the system). Hence, there are no good preconditions to charge the players for recommendations, as we did in another treatment. Consequently, only a few players requested recommendations, which reduced their reliability, so that the overall performance of the system decreased.

5. Master equation description of iterated decisions

The stochastic description of decisions that are taken at discrete time steps (e.g. on a day-to-day basis) is possible by means of the time-discrete master equation [2]

$$P(\boldsymbol{n}, t + \Delta t) = \sum_{\boldsymbol{n'}} P(\boldsymbol{n}, t + \Delta t | \boldsymbol{n'}, t) P(\boldsymbol{n'}, t) \tag{11}$$

with $\Delta t = 1$ unit time. Herein, $P(\boldsymbol{n}, t)$ denotes the occurrence probability of the *configuration* $\boldsymbol{n} = (n_1, n_2)$ at time t. This vector comprises the *occupation numbers* n_i and reflects the decision distribution in the system. As the number of individuals changing to the other alternative is given by a *binomial distribution*, we obtain the following expression for the *configurational transition probability*:

$$
\begin{aligned}
P((n_1, n_2), &t + 1 | (n_1 - \Delta n_1, n_2 + \Delta n_1), t) \\
&= \sum_{k=0}^{\min(n_1 - \Delta n_1, n_2)} \binom{n_2 + \Delta n_1}{\Delta n_1 + k} \\
&\quad \times p(1|2, n_1 - \Delta n_1; t)^{\Delta n_1 + k} [1 - p(1|2, n_1 - \Delta n_1; t)]^{n_2 - k} \\
&\quad \times \binom{n_1 - \Delta n_1}{k} p(2|1, n_1 - \Delta n_1; t)^k [1 - p(2|1, n_1 - \Delta n_1; t)]^{n_1 - \Delta n_1 - k}. \tag{12}
\end{aligned}
$$

This formula sums up the probabilities that $\Delta n_1 + k$ of $n_2 + \Delta n_1$ previous 2-choosers change independently to alternative 1 with probability $p(1|2, n_1 - \Delta n_1; t)$, while k of the $n_1 - \Delta n_1$ previous 1-choosers change to alternative 2 with probability $p(2|1, n_1 - \Delta n_1; t)$, so that the net number of changes is Δn_1. If $\Delta n_1 < 0$, the roles of alternatives 1 and 2 have to be interchanged. Formulae (11) and (12) would look even more complicated if we distinguished several characters a. We would, then, have to replace the binomial distributions by multinomial ones.

The potential use of equation (12) is the calculation of the statistical variation of the decision distribution or, equivalently, the number n_1 of 1-choosers. It also allows one to determine the variance, which the optimal guidance strategy should minimize in favour of reliable recommendations.

6. Summary and outlook

In this work, we have discovered that the dynamics of iterated decisions based on aggregate information is intermittent. In order to control intermittency, we have developed a stochastic description and a nonlinear feedback mechanism. That is, the application of several physical concepts and methods allowed us to gain a detailed understanding of decision dynamics, which is required for practical applications.

In more detail, we have explored different and identified superior ways of information presentation that facilitate to guide user decisions in the spirit of higher payoffs. By far the

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least standard deviations from the user equilibrium could be reached by presenting the own payoff and the potential payoff, if the respective participant (or a certain fraction of players) had additionally chosen the other alternative. Interestingly, the decision dynamics were found to be intermittent, similar to the volatility clustering in stock markets, where individuals also react to aggregate information. This results from the desire to reach above-average payoffs, combined with the immanent overreaction in the system. We have also demonstrated that payoff losses due to a volatile decision dynamics (e.g., excess travel times) can be reduced via user-specific recommendations by a factor of three or more. Such kinds of results will be applied to the route guidance on German highways (see, for example, the project SURVIVE conducted by Nobel prize winner Reinhard Selten and Michael Schreckenberg). Optimal recommendations to reach the user equilibrium follow directly from the derived balance equations (6) and (7) for decision changes based on empirical transition and compliance probabilities. The quantification of the transition probabilities needs a novel stochastic description of the decision behaviour, which is not just driven by the potential (gains in) payoffs, in contrast to intuition and established models. To understand these findings, one has to take into account individual learning.

Obviously, it requires both theoretical and experimental efforts to get ahead in decision theory. In a decade from now, the theory of 'elementary' human interactions will probably have been developed to a degree that allows one to systematically derive social patterns and economic dynamics on this ground in a similar way as the structure, properties and dynamics of matter have been derived from elementary physical interactions. This will not only yield a deeper understanding of socio-economic systems, but also help to more efficiently distribute scarce resources such as road capacities, time, space, money, energy, goods, or our natural environment. One day similar guidance strategies as the ones suggested above may help politicians and managers to stabilize economic markets, to increase average *and* individual profits, and to decrease the unemployment rate. Physics can contribute to this goal, in particular with the methods developed in the fields of nonlinear dynamics and statistical physics.

Acknowledgments

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