# Computer simulations for a fractional calculus derived internet traffic model

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## Abstract

In modern communication networks it is interesting not only to know how fast packets can travel in the network, another parameter object of study is the number of information packets at each node of the network at time t. Some models of traffic in imperfect communication Networks has been proposed for the Internet, important features such as delay in transmission and packet lost and congestion at the nodes must be taken in consideration by the model. Some imperfect traffic models based on fractional calculus have been proposed. This works present some computer simulations of internet traffic and comprare the results with the ones provided by the fractional calculus models.

**Keywords:** Internet, traffic model, fractional calculus, packet delay.

# 1 Introduction

In the past few years, growth in the field of communication networks [1, 2, 3], multiagent systems [4] and complex systems [5] have generated many studies focused on the behavior of network packages transmission. In modern communication networks as, for example Internet, the temporal evolution of the transmission time between two fixed machines in the network it is known to be a chaotic time series of a dynamical system with positive Lyapunov exponents. Even under this limitation, some parameters, such as, the macroscopic behavior of the global Internet network response times can be characterized and predicted by the use of artificial neural networks [6]. As pointed out in [7], performance parameters associated with a telecommunication network warrants a meaningful model for the complexity of the communication system.

A relevant modeling of such complexity should include stochasticity of interacting resources and in the flow of information between the nodes of the network. In Internet, for example, there exists a delay between the time when a node receives a package and the time when this package is sent to a neighbor node. This delay is due mainly to the managing, storing, and queuing of the package as well as to saturation of neighbor nodes.

In Internet not every package sent through the network reaches its destination, noisy channels produce that some packages become not valid and therefore are thrown away from the network. In disease transmissions not every contact among individuals produces the transmission of the disease. These reasons point to the fact that the existing models with instant and flawless transmissions are no longer able to predict the response time in networks with time delay and faulty transmissions and, therefore, models that reflect these properties need to be taken in account [8, 9]. In [10] the delay of packets transmission between two given nodes of a network is studied when each node maintains a queue of packets and the delay in sending each packet from one node to the following in its route is a random variable that follows an uniform distribution.

Some Network traffic characteristics are more efficiently described in terms of fractal than conventional stochastic processes. Long–range dependency in statistical moments have been found in corporate, local and wide-area networks [11, 12]. In these works, the correlation function of network process decays much more slowly than exponential, showing in fact a power–law decay. This behavior has been explained on the basis of diffusion processes that describe the evolution o a system with properties of information loss. This power–law dependence can be explained under the light of fracionary calculus and fractional derivative equations [13]. In this framework, some models of Internet traffic has been developed on the basis of fractional calculus. For example in [14] the most probable number of packets in the site $x$ at the moment $t$, $n(x,t)$ is estimated under the assumption that the time delay of packet transmission between two given nodes is given by a probability density of the form:

$$f(t) = \frac{\gamma}{(1+t)^{(1+\gamma)}} \tag{1}$$

where the packet loss situation corresponds to a condition that the packet will not leave one of the intermediate nodes in its route. Even when these works present interesting analytical results, there is no a comparison between these analytical results and the behavior observed in real data or in data obtained by computer simulations.

In this work we explore, by means of computer simulations, the behavior of a traffic model that present a time delay in packet transmission that follows a probability density given by eq. (1).

## 2 Theoretical Framework

In [14] a theoretical model for network traffic is developed when the probability distribution function follows the expression given by 1. For this distribution the most probable number of packets at node $x$ at time $t$ $n(x,t)$ is given by the expression:

$$n(x,t) = \int_0^t n(x-1, t-\tau) f(\tau) d\tau + n(x,0) F(t) \tag{2}$$

where $F(t) = 1 - \int_0^t f(\tau) d\tau$.

With these designations, the equation of package migration can be expressed as a fractional derivative equation of the form:

$$\Gamma(1-\gamma) D_t^{\gamma}[n(x,t)] = -\frac{\partial n(x,t)}{\partial x} + \frac{n(x,0)}{t^{\gamma}} \tag{3}$$

If we assume the initial conditions $n(0,0) = n_0$ and $n(k,0) = 0 \ \forall \ k \neq 0$ it can be obtained for the discrete case:

$$n(x,t) = n_0 \left\{ \frac{1}{t^{\gamma}} - x \left[ \frac{\Gamma^2(1-\gamma)}{\Gamma(1-2\gamma)} \cdot \frac{1}{t^{2\gamma}} + \frac{\Gamma(1-\gamma)}{\Gamma(-\gamma)} \cdot \frac{1}{t^{\gamma+1}} \right] \right\} \tag{4}$$

and finally:

$$c(x,t) = n_0^2 \Gamma(1-\gamma) t^{1-2\gamma} \left[ \frac{1}{\Gamma(2-2\gamma)} - x \cdot \frac{\Gamma(1-\gamma)}{\Gamma(2-3\gamma)} \cdot \frac{1}{t^{\gamma}} \right] \tag{5}$$

where $c(x,t)$ is the correlation function.

## 3 Point-to-point transport (unicast)

In unicast or point-to-point transmission, a set of packages has to cross a number of intermediate machines (routers) in order to go from the source to the destination. Different path lengths are possible. In our unicast experiment, each node maintains a queue of packages. One source node sends a file of $l$ packages to a destination node by a route of $r$ intermediate step nodes. Each node in the route maintains a queue of packages. Each package has a time counter that represents the time that rests to send the package to the following node.

The following happens at each time instant for each node.

- The node sends all the packages that are ready (time counter equal 0) to the following node in the route.
- The node decrements by one the time counter of all the packages remaining in its queue.

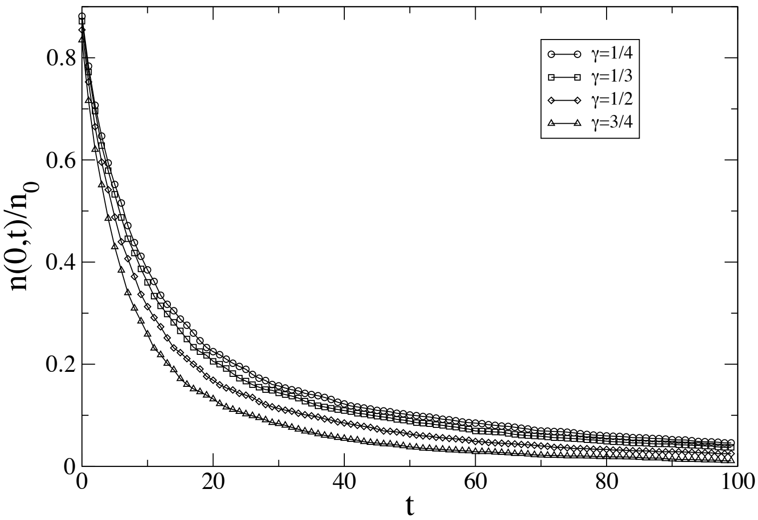


Figure 1: Normaliced number of packages at node 0 $n(0,t)$. The path consists of 100 nodes and $n_0 = 3000$

- The node receives a set of packages from the previous node in the route and assigns a time delay to the package following a given probability distribution $f(t)$.

At each time step $t$ the number of packages $n(x,t)$ at node $x$ is exactly the size the node $x$ queue.

From eq. (5) follows:

$$n(0,t) = n_0 \left\{ \frac{1}{t^\gamma} \right\} \tag{6}$$

This same behavior can be observed in our simulations. See fig 1. The number of packages at each node defined by eq. 4 presents also a similar behavior in our simulations.

In figures 3 and 4 the behavior of the correlation $c(m,\tau) = \sum_{k=0}^{N} \sum_{t=0}^{T} n(k,t)n(k+m;t-\tau)dt$ and $D(t) = c(0,t)$ is plotted. As expected, the correlation functions present a decay concerning the value of $t$.

# 4 Conclusion

A set of computer simulations have been performed to test the results of a model of Internet traffic based on fractional calculus. The computer simulations show a similar behavior a similar behavior to the fractional model with respect to the number of packages at each node. The computer simulations also present a decay in the covariance, as should be expected, in difference with the fractional model presented at [14].

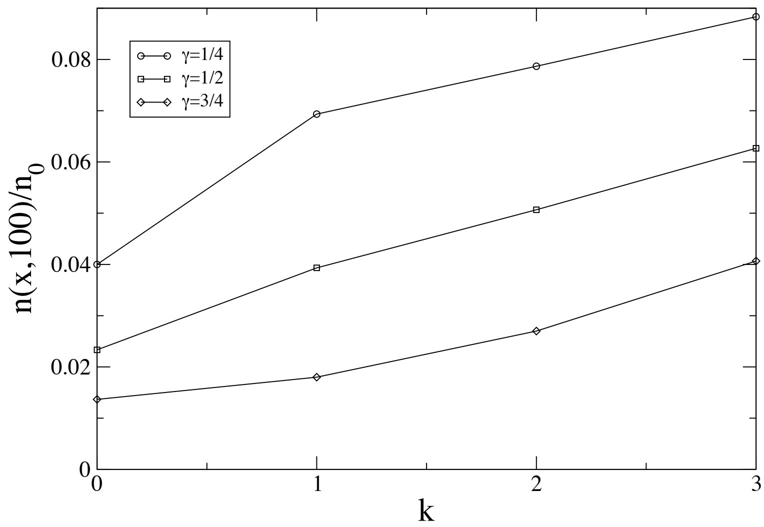


Figure 2: Normaliced number of packages at node $x$ and $t = 100$ $n(x, 100)$. The path consists of 100 nodes and $n_0 = 3000$

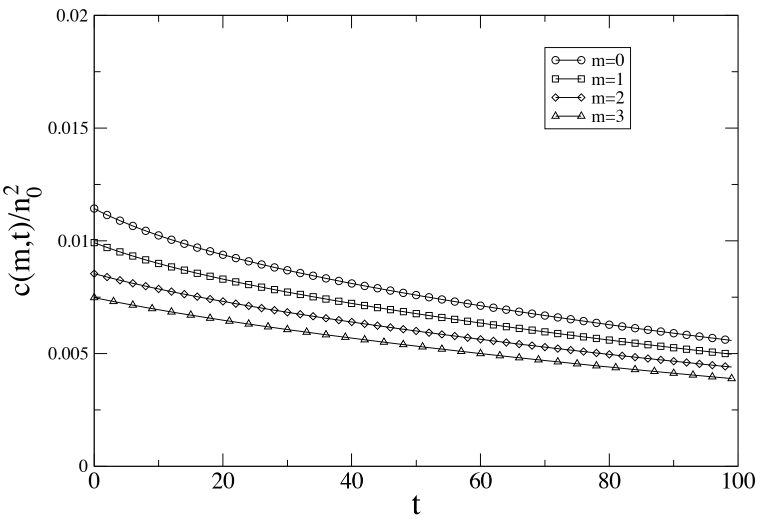


Figure 3: Covariance at node $m$ for $\gamma = \frac{1}{4}$. The path consists of 100 nodes, $n_0 = 3000$ and $T = 500$

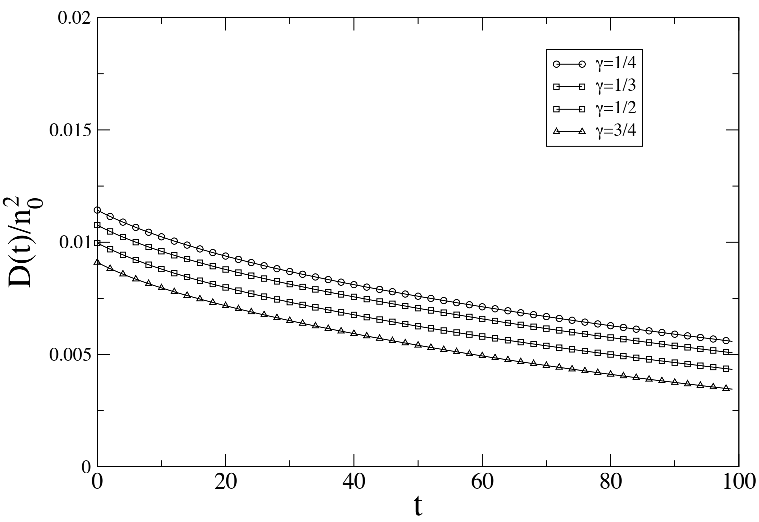


Figure 4: Covariance at node $0$ for different values of $\gamma$. The path consists of 100 nodes, $n_0 = 3000$ and $T = 500$

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# References


[1] B. Cheswick and H. Burch,*"Mapping the Internet"*, Computer, 32(4), 97, 1999.

[2] K. Claffy, *"CAIDA: Visualizing the Internet"*, Internet Computing Online, http://computer.org/internet/v5n1/caida.html 2001.

[3] K. Claffy, *"Measuring the Internet"*, IEEE Internet Computing, 4(1), 73–75, 2000.

[4] F. Corbacho and M. A. Arbib, *"Schema-based Learning: Towards a Theory of Organization for Adaptive Autonomous Agents"*, Proceedings of the First International Conference on Autonomous Agents, Santa Monica, CA, 1997, ACM, New York, 1997.

[5] C. Aguirre and F. Corbacho and R. Huerta, *"Static and Dynamic Properties of Small-World Netwoks Connection Topologies Based on Transit-stub Networks"*, Complex Systems, 14(1), 1–28, 2003.

[6] J. C. Park, *"Chaos and Predictability of Internet Transmission Times"*, Complex Systems, 12, 297–316, 2000.

[7] P. M. Gade and C.-K Hu, *"A Complex Systems Characterization of Modern Telecommunication Services"*, Complex Systems, 12, 31–69, 2000.

[8] X. S. Yang, *"Chaos in small-world networks"*, Phys. Rev. E, 63, 462061–64, 2001.

[9] C. Aguirre, P. Pascual D. Campos and L. Vázquez, *"Chaotic behavior of small-world communication networks with imperfect transmission"*, The 10th IFAC/IFORS/IMACS/IFIP Symposium on Large Scale Systems: Theory and Applications, LSS2004, Osaka, Japan, IFAC Proceedings 2004.

[10] C. Aguirre and J. Martinez-Munoz and F. Corbacho and R. Huerta, *"Small-World Topology for Multi-Agent Collaboration"*, Proceedings of the 11th International Workshop on Database and Expert Systems Applications, 231–235, 2000.

[11] W.E. Leland , M.S. Taggu, Willinger and D.V. Wilson, *"On the Self-Similar Nature of Ethernet Traffic"*. Proceedings of ACM SIGCOMM'93, San Francisco, 1993.

[12] S.M. Klivansky, A. Mukherjee and C. Song, *"On Long-Range Dependence in NSFNET Traffic"*, Technical Report CIT-CC-94-61, 1994.

[13] A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *"Theory and Applications of Fractional Differential Equations"*, Elsevier, 2006.

[14] V. Zaborovski, Y.Podgurski and S.Yegorov, *New traffic model on the base of fractional calculus*, Internet http://www.neva.ru/conf/art/art8.html