Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 27 October 2000 (MN LaTeX style file v1.4)

An observational constraint on gravitational lensing by objects of mass $10^{9.5}$–$10^{10.9}$ $\mathbf{M_\odot}$

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27 October 2000

ABSTRACT

A radio-based search for strong gravitational lensing, with image separations in the range 160–300 milliarcsec (mas), has yielded a null result for a sample of 1665 sources (Augusto, Wilkinson & Browne 1998) whose mean redshift is estimated to be ~ 1.3. The lensing rate for this previously-unexplored separation range, $< 1 : 555$ at the 95% confidence level, is less than on arcsecond-scales—as expected from models of lensing galaxy populations. Lensing on 160–300 mas scales is expected to arise predominantly from spiral galaxies at a rate dependent on the disk-halo mass ratio and the evolving number density of the population with redshift. While the present sample is too small for there to be a high probability of finding spiral galaxy lenses, our work is a pilot survey for a much larger search based on the full CLASS database which would provide useful information on galactic structure at $z \sim 0.5$. We examine other possible lens populations relevant to our present search, in particular dwarf galaxies and supermassive black holes in galactic nuclei, and conclude that none of them are likely to be detected. Our null result enables us formally to rule out a cosmologically significant population of uniformly-distributed compact objects: $\Omega_{\rm CO} < 0.1$ (95% confidence) in the mass range $10^{9.5}$–$10^{10.9}$ $M_\odot$.

Key words: galaxies: compact - general ; cosmology: dark matter - gravitational lensing;

1 INTRODUCTION

Systematic radio-based surveys have proved to be a very successful way of finding gravitational lens systems with image separations in the range 0.3–6 arcsec; the lensing masses are the central regions of normal galaxies. The Jodrell-VLA Astrometric Survey (JVAS)/Cosmic Lens All-Sky Survey (CLASS) surveyed over 12000 sources, using the VLA, MERLIN and the VLBA as successive high resolution filters. These have, so far, found 19 such lens systems (Browne et al. 2000 and unpublished). In the course of a general investigation of the properties of flat-spectrum radio sources, drawn principally from the JVAS, Augusto, Wilkinson & Browne (1998; paper I hereafter) carried out a search for multiple imaging with smaller separations in the range ∼90–300 mas. Using essentially the same VLA, MERLIN and VLBA filtering process employed in the main JVAS/CLASS lens surveys, no cases of multiple imaging were found in a sample of 1665 sources. In this paper we examine the implications of this null result.

The classic statistical study of gravitational lensing probabilities was carried out by Turner, Ostriker & Gott (1984; TOG84 hereafter) who calculated the distribution of the angular separations of multiple images expected from normal galaxies. TOG84 predicted that $\sim 88\%$ of lenses associated with normal galaxies should occur in the range 0.3–6 arcsec. All of the known cases of strong lensing by individual galaxies fall in this angular size range, with a mean separation consistent with TOG84's expectation of ~ 1.5 arcsec (Browne et al. 2000) and a smallest separation of 0.335 arcsec (Patnaik et al. 1993). It is important, however, to subject the TOG84 predictions to further observational scrutiny in order to see whether there are other contributions to the lens population. Our results are for a previously unexplored separation range and hence provide a test for lower-mass galaxies (or components of galaxies) than were considered by TOG84. This is particularly relevant since recent calculations of the lensing contribution of spiral galaxies suggest that TOG84 may have underestimated their contribution to the lensing optical depth by more than a factor two (see section 2.1).

Our results also enable us to place observational constraints on 'non-standard' populations of potential lensing masses. Any mass concentration with size $\lesssim 0.02\sqrt{M/M_\odot}$

pc is 'compact' as regards gravitational lensing (Press & Gunn 1973) and unseen dark Compact Objects (CO) could produce lensing on scales of interest for this paper. If present in large enough numbers, such CO could contribute to the solution of the dark matter problem. In the extreme case of a universe filled with a critical density of CO of a particular mass ($\Omega_{\rm CO} = 1$), the odds are even that any object at $z = 2$ will be multiply imaged into two images with a brightness ratio $< 10 : 1$ (Press & Gunn 1973). The lensing probability scales directly with the number density of CO and hence the statistics of gravitational lensing in surveys of distant sources provide direct constraints on $\Omega_{\rm CO}$. Throughout this paper we take the value of H_0 to be 65 km s^{-1} Mpc^{-1}.

2 LENSES OF MASS $\sim 10^{9.5}$–$10^{10.9}$ M$_\odot$

Our basic observational result is that amongst a parent sample of 1665 flat-spectrum radio sources there are no cases of gravitational lensing with separations in the range ∼90–300 mas. The only weak candidate left from paper I, B0225+187, was ruled out from a subsequent MERLIN+EVN 1.6 GHz map (Augusto, unpublished). For our search, the flux-ratio (R) of candidate images is dependent on their angular separation (δ) since this is comparable to the beam size of the VLA in A configuration at 8.4 GHz (∼ 200 mas). Fig. 1, reproduced from paper I and discussed in detail there, shows the angular selection function for our search derived from extensive simulations with the VLA 'snapshot' aperture-plane coverage and a range of source types, position angles and declinations. We were conservative in deriving this selection function, which represents the 'lensing filter' at essentially 100% confidence. Sources with smaller separations could be detected, but less often, depending on their position angle. For example, an equal double source ($R = 1$) with a separation of 50 mas had a ∼ 50% chance of being included and one with 75 mas separation had ∼ 97%. However, in order to simplify our analysis and its interpretation, we will only consider separations in the range 160–300 mas where we are confident (Fig. 1) of selecting sources with $R \sim 7$—the usual definition of strong lensing (e.g. TOG84).

We can, therefore, place a formal upper limit on the lensing rate in the range 160–300 mas using the equation

$$P(0, N) = (1 - p)^N, \qquad (1)$$

where $P(0, N)$ is the probability of finding no cases of multiple imaging amongst N sources (1665 in our case) when the probability of lensing an individual source is p. To obtain an upper limit to p at the 95% confidence level we require $P(0, N) \le 0.05$ and hence obtain $p = 0.00180$. The upper limit ($1/p$) on the strong-lensing rate for image separations 160–300 mas is, therefore, 1 : 555 (95% confidence). We now examine the implications of this result for various possible lensing objects.

2.1 Normal galaxies

Since the parent sample for our lens search is dominated by JVAS sources (Augusto 1996) we can directly compare our null result with the lensing rate at arcsec-scale separations found in 2384 JVAS sources (King et al. 1999). Since five

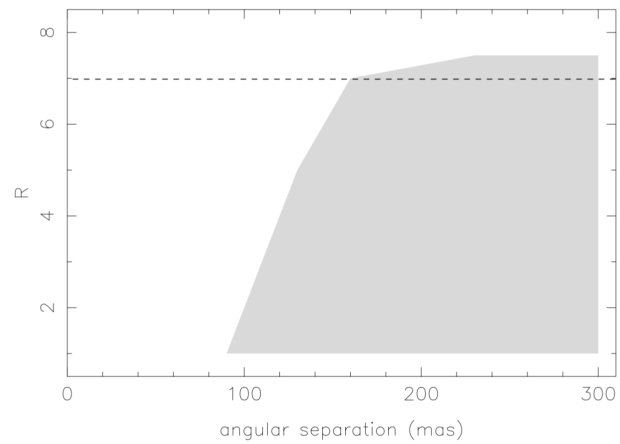

Figure 1. The angular separations (δ) of the multiple compact components whose maximum flux density ratio is R, which will be selected from our VLA A-array 'snapshot' data at 8.4 GHz using the criterion of a ∼ 25% decrease in correlated flux density from the shortest to the longest baselines. In the shaded region lens candidates will be picked out with near 100% reliability (reproduced from paper I). In order to simplify our analysis we only considered the range $160 \le \delta \le 300$ mas where we are confident of selecting sources with $R \le 7$.

suitable⋆ JVAS lenses were found, the rate is ∼ 1 : 478. Our null result shows that, as predicted by TOG84, the lensing rate is almost certainly lower for image separations of ∼ 0.2 arcsec than it is for arcsec-scale separations. Taking the argument one step further, from TOG84's probability distribution we estimate that only ∼ 6% of lensing by normal galaxies should produce image separations in the range 160–300 mas, i.e., the lensing rate should be about 1:7000.

The dominant contribution to the overall gravitational lensing cross-section in TOG84's models is by elliptical galaxies; spiral galaxies are expected to contribute only ∼ 20% of the total. This is consistent with the known population of arcsec-scale lenses which only contain a few spirals. However, since TOG84 published their paper, there has been considerable further work on the lensing properties of disks embedded in dark matter halos (e.g. Keeton & Kochanek 1998; Bartelmann & Loeb 1998; Blain, Möller & Maller 1999; Bartelmann 2000). The cross-sections are dominated by edge-on disks and the predicted lensing rate is dependent on the balance between the disk and halo masses. Keeton & Kochanek (1998) predict that, when averaged over all inclinations, there should be little change in the contribution of spirals compared with the predictions of TOG84. In contrast the other models (Blain et al. 1999; Bartelmann & Loeb 1998; Bartelmann 2000), which involve maximal disks and also consider the effect of evolution of the spiral population with redshift, predict significant enhancements of the total spiral fraction, by factors of two or more, compared with TOG84.

Since spirals contribute most at small image separations, the enhancement over TOG84 could become large for

⋆ One of the JVAS lenses (B1030+074) has $R \sim 15$ and hence has not been included in our estimate of the lensing rate which is based on JVAS lenses with $R \le 7$.

the previously unexplored separations to which our search is sensitive. For example, Bartelmann (priv. comm.) estimates that between 10% and 20% of *all* galaxy-mass lenses could have separations in the range 0.1–0.3 arcsec as opposed to $\sim 8\%$ in TOG84. These models suggest, therefore, that the lensing rate appropriate to our survey is one in a few thousand background objects searched. Thus our 1665-source parent sample still does not have a high probability of containing a small-separation spiral lens. However, an order-of-magnitude larger search would place significant contraints on the uncertain disk/halo mass ratio in spiral galaxies at $z \sim 0.5$. We return to this point later.

2.2 Dwarf Galaxies

TOG84 did not consider lensing by dwarf galaxies. However, our search is sensitive to the upper end of the dwarf mass range and so we must ask the question: are the properties of the dwarf galaxy population such as to make multiple imaging likely in the present context? There are two issues to address: (i) are dwarf galaxies compact enough to have the critical surface mass density (~ 1 g cm^{-2}) for multiple imaging? and (ii) are there enough of them to compensate for their much smaller lensing cross-section compared with normal ellipticals? We will address these issues briefly in turn.

The mass distributions of dwarf galaxies are in general poorly known. However, from a study of published HI rotation curves of nine, gas-rich, dwarf irregulars (dI) Augusto (1996) showed that their mean surface mass densities are one to two orders of magnitude less than those of the normal ellipticals which dominate the lensing statistics. On this basis dI can be ruled out as potential lenses. The surface mass densities of gas-poor dwarf ellipticals (dE) are essentially unknown but since most show compact nuclei (e.g. Binggeli & Cameron 1991; Ferguson & Sandage 1989; Vader & Sandage 1991) it is possible that dE do have the requisite surface mass density for multiple imaging. The remaining types of dwarf galaxies are a lot less common.

Better determinations of the space density of dwarf galaxies have recently become available. For example, from studies of the Hubble Deep Field, Driver (1999) shows that while dwarf galaxies at $0.3 < z < 0.5$ (which are well-placed for lensing) are more numerous than giant galaxies, the excess is only by factors of order unity. Essentially the same conclusion is reached by Loveday (1997) from independent ground-based data. Driver (1999) goes on to show that, for reasonable assumptions about their mass-to-light ratios, dwarf galaxies only account for $\sim 16\%$ of the total galaxy mass budget.

Since the separation range to which our search is sensitive is 5–10 times smaller than the typical separation of lens systems in JVAS the corresponding lensing cross-section is 25–100 times smaller. We conclude, therefore, that even if *all* dE are capable of multiple imaging, on current estimates their numbers are too small, by at least an order-of-magnitude, to produce a significant number of lensing events in a sample of 1665 objects.

2.3 Dark Compact Objects

2.3.1 Supermassive black holes in galaxies

We can be reasonably certain of one population of dark compact objects relevant to the present search—the supermassive black holes (SBH) which power active galactic nuclei (AGN) and which may well lie hidden in many inactive galactic nuclei (IGN). Quasars are most likely to host a $\sim 10^9$ M$_\odot$ SBH but their average separation on the sky for $z < 1$ (appropriate for lensing) is ~ 1000 arcsec (e.g. the 2dF survey at *www.mso.anu.edu.au/ rsmith/QSO_Survey*) and hence, for an Einstein radius of ~ 0.1 arcsec, the lensing probability for background sources is very low ($\sim 10^{-8}$). Even in the limit that all normal galaxies harbour 'dead' quasars and hence contain a SBH (e.g. Magorrian et al. 1998), and even if all these SBHs have masses $\sim 10^9$ M$_\odot$, their associated lensing rate must be two orders of magnitude below the JVAS rate for arcsec-scale lensing simply because of the smaller cross-section. We cannot, therefore, place a useful constraint on the numbers of SBH in IGN from our present results, nor from any future search of a ten-times larger parent sample.

2.3.2 Relic supermassive black holes

It has long been been suggested (e.g. Press & Gunn (1973); Carr, Bond & Arnett (1984) and references therein) that a population of black holes could have formed early in the history of the universe and could remain in existence, perhaps providing seeds for AGN (e.g. Fukugita & Turner 1996). Turner & Umemura (1997) suggested that relic SBH might be detected by means of occasional very strong gravitational lensing of luminous stars in distant galaxies but, as was first suggested by Press & Gunn (1973) it is high resolution radio imaging which allows the most direct search for lensing by relic SBH. Fukugita & Turner (1996) estimate that the number density of SBH of mass $\sim 10^9$ M$_\odot$ could be comparable with that of luminous galaxies and up to three orders of magnitude higher than the peak number density of quasars. But even so, from the simple cross-section arguments, it is clear that much larger samples than the present one will be needed to put the hypothesis of a relic SBH population to a severe observational test.

2.3.3 Dark galaxies

Peebles (1968) proposed that a large number of 'dead galaxies' could solve the dark matter problem while more recently it has been proposed that very massive (10^{12}–10^{13} M$_\odot$) dark objects could give rise to the quasar pairs with separations ~ 10 arcsec by gravitational lensing (Hawkings 1997). There is, however, no supporting evidence for such a population of massive dark objects. Kochanek, Falco and Munoz (1999) argue that a comparison of the radio and optical properties of the pairs rules out the massive lens hypothesis. Furthermore, HST imaging of confirmed arcsec-scale lenses found in the JVAS/CLASS surveys always shows a lensing galaxy with a relatively normal mass-to-light ratio between the images (Jackson et al. 1998).

There is a strong increase with redshift of the fraction of morphologically peculiar galaxies (see the review by Ellis (2000) and references therein). It seems most likely that many of the distant blue irregular galaxies are transformed by mergers into normal ellipticals and spirals. It is, however, possible that they could be seen at an unusually active period in their history and that, starved of infalling gas, they could have simply faded away to low surface brightness (LSB) systems which would be hard to detect (e.g. Ellis 2000). However, without a model of the remaining LSB systems fraction and their likely mass distribution, one cannot make a prediction as to the likelihood of detecting them by their lensing effects.

Although there is no evidence for them in arcsecond-scale lens searches, dark galaxies with smaller masses of $\sim 10^{9.5}$–$10^{10.9}$ $M_\odot$ (and respective upper limits on their sizes of $\sim$ 1.2–5.6 kpc) could exist and would produce multiple images of background sources in the range 160–300 mas. For these we can place the same cosmological density limits as for compact objects (e.g. black holes) and this is the subject of the next section.

2.4 Quantitative limits on Ω_{CO}

Despite the apparent lack of lensing objects which would produce 160–300 mas image separations with a significant probability in our 1665-source sample, it is still useful to set quantitative limits on Ω_{CO} based on our null result. To do this we adopt the 'detection volume' method introduced by Nemiroff (1989). Kassiola, Kovner & Blandford (1991) made specific calculations for radio-surveys and hence these directly apply to our case. The 'detection volume' method consists of three steps, once a survey is complete: i) derive the volume between each source in the survey and the observer ('detection volume'); ii) add up the 'detection volumes' for all the sources in the survey; iii) compare the resultant total volume with the volume expected for a given value of Ω_{CO}.

In carrying out this calculation we make the following assumptions: i) an Einstein-de-Sitter universe ($\Omega_0 = 1$; $\lambda_0 = 0$); ii) no magnification bias; iii) $\Omega_{CO} \ll 1$. For our present purpose assumption (i) is conservative. If the geometry of the universe is in part determined by the cosmological constant, then the expected number of lenses in a given sample rises (e.g. Fukugita et al. 1992). A null lensing result therefore implies a stronger limit on the number density of lensing objects on such a Universe. Assumption (ii) is also conservative. Due to magnification by gravitational lensing, flux-limited samples contain intrinsically less luminous sources than the flux density threshold would normally allow (see review on Narayan & Wallington 1993). For flat-spectrum radio sources, however, the magnification bias is expected to be relatively weak (King & Browne 1996) and we have therefore not taken it into account. If there is any bias our null result constitutes a stronger constraint on Ω_{CO}, since the actual number of sources in the parent sample would increase. Assumption (iii) allows us to ignore collective lensing effects of the sort explored by Kassiola et al. (1991).

Following Kassiola et al. (1991), for a point mass lens and a background source of a given redshift, we calculate the probability of *not* detecting primary and secondary images with a flux ratio smaller than R, the threshold at which the selection process stops reliably picking lens candidates. The 'detection volume' formulae presented in Kassiola et al. (1991) are truncated because of the observational limits of a survey. Thus there is a limit for the 'resolution' of the instrument used (δ) and also for the maximum image separation allowed for the lens candidates (Δ). Following Kassiola et al. (1991), we have for the truncated detection volume:

$$V_t = \frac{16\pi GM}{H_0^2}(R^{1/4} - R^{-1/4})^2 \nu(z_s, M, \delta, \Delta)$$

with M the mass of the point lens, z_s the redshift of the background source and ν a dimensionless volume function. While ν is a function of mass, δ and Δ effectively define lower and upper limits of a mass range for which ν is approximately constant and the truncation is small. Outside this mass range truncation increases sharply and ν (and V_t) rapidly tends to zero. For the mass range $M_1 < M < M_2$ the limit on $\Omega_{CO}^{M_1-M_2}$ at the 100Π% confidence level is obtained from:

$$\Omega_{CO}^{M_1-M_2} < \frac{-\ln(1-\Pi)}{6\,(R^{1/4} - R^{-1/4})^2 \sum_i \nu(z_{s_i}, M, \delta, \Delta)}. \qquad (2)$$

Our search parameters appropriate for equation 2 are: $\delta = 160$ mas; $\Delta = 300$ mas and $R = 7$. These are conservative parameters derived from the complicated $R(\delta)$ function of Fig. 1. Augusto (1996) has shown that the results obtained by including the range 90–160 mas are similar. To determine $\sum_i \nu$, we need redshift information, but this is very limited for the z_{s_i} of the 1665-source parent sample. We can, however, make use of the 90% complete redshift information of the flux-limited 293-source CJF sample (Taylor et al. 1996), which is drawn largely from the JVAS catalogue. The lower-limiting flux density of the CJF sources is 350 mJy at 5 GHz compared with $\sim$ 130 mJy for our sample. There is, however, no significant variation in the redshift distributions and the mean redshift ($\sim$ 1.3) of flat-spectrum radio sources in the range 1 Jy to $\sim$ 30 mJy (e.g. Falco, Kochanek & Munoz 1998; Marlow et al. 2000) and so little error will arise from applying the well-defined CJF statistics to our sample. The formal constraints on Ω_{CO} at the 95% confidence level are then shown in Figure 2 and they are $\Omega_{CO}^{9.5-10.9} < 0.10$.

3 DISCUSSION

Among a sample of 1665 flat-spectrum radio sources, with an assumed mean redshift of $\sim$ 1.3, no multiple images were found with separations in the range 160–300 mas (paper I and Augusto, unpublished). The lensing rate for compact masses in the range $\sim 10^{9.5}$–$10^{10.9}$ $M_\odot$ is therefore less than 1 : 555 at 95% confidence. This result does not contradict TOG84 predictions based on the properties of normal E/S galaxies. No unexpected population of small galaxies has thus been detected and it seems that, despite their large numbers, $\sim 10^9$ $M_\odot$ dwarf galaxies are not contributing significantly to multiple imaging on 160-300 mas. This is consistent with the current knowledge about their numbers and mass distributions.

Recent work on the contribution of $z \sim 0.5$ spirals to the overall lensing statistics gives contradictory results: there could be significant (greater than a factor of two) enhancements of the total spiral fractional contribution. Part of the

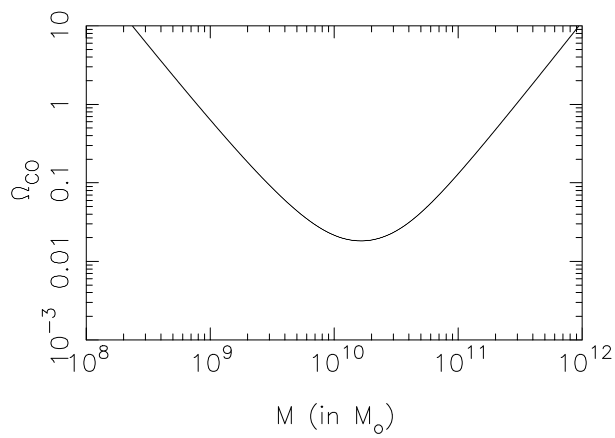

Figure 2. The limits on Ω_{CO} at the 95% confidence level from the failure to detect multiple imaging amongst 1665 flat-spectrum sources with a mean redshift of 1.3. These are $\Omega_{CO}^{9.5-10.9} < 0.10$ (on the $\sim 10^{9.5}$–$10^{10.9}$ $M_\odot$ mass range), $\Omega_{CO}^{9.9-10.6} < 0.03$ and $\Omega_{CO}^{10.2} < 0.02$ (strongest constraint) for objects with mass $\sim 10^{10.2}$ $M_\odot$.

motivation for this new work on the spiral population has been to anticipate the capabilities of ALMA and NGST at ∼0.1 arcsec resolution in the sub-mm and infra-red bands. Radio-based surveys are similarly well-suited to an unbiased search for spiral galaxy lensing since they are also not affected by dust obscuration in edge-on disk systems. It is now possible to mount a significant test of this prediction, since by using the same methodology as adopted for Paper I we can look for compact lenses in the now-completed CLASS surveys. Since the combined JVAS/CLASS surveys have, so far, found 19 lenses with separations 0.3–6 arcsec in a total sample of ∼ 12000 sources, a search in the range 0.1-0.3 arcsec using the same parent sample should find some small-separation lenses (likely to be spirals). The results would place significant contraints on the uncertain properties of spiral galaxies at the redshifts appropriate to multiple imaging ($z \sim 0.5$).

Analysing the simpler point mass lens constraints, we conclude that the cosmological density of compact objects (which include black holes and ∼1–5 kpc compact galaxies) in the mass range $\sim 3 \times 10^9$ to $\sim 8 \times 10^{10}$ $M_\odot$, has to be $\Omega_{CO}^{9.5-10.9} < 0.10$ (95% confidence level). From our new result, it seems that $10^{9.5}$–$10^{10.9}$ $M_\odot$ SBH in this range cannot contribute significantly to solving the dark matter problem.

ACKNOWLEDGMENTS

Pedro Augusto thanks the Fundação para a Ciência e a Tecnologia for the grant Ciência/PraxisXXI/BD2623/93-RM. We acknowledge many discussions with Ian Browne and Shude Mao and thank Matthias Bartelmann for a specific discussion on spiral lens statistics.

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