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Measurement Uncertainty of Dew-Point Temperature in a Two-Pressure Humidity Generator

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Abstract This article describes the measurement uncertainty evaluation of the dew-point temperature when using a two-pressure humidity generator as a reference standard. The estimation of the dew-point temperature involves the solution of a non-linear equation for which iterative solution techniques, such as the Newton–Raphson method, are required. Previous studies have already been carried out using the GUM method and the Monte Carlo method but have not discussed the impact of the approximate numerical method used to provide the temperature estimation. One of the aims of this article is to take this approximation into account. Following the guidelines presented in the GUM Supplement 1, two alternative approaches can be developed: the forward measurement uncertainty propagation by the Monte Carlo method when using the Newton–Raphson numerical procedure; and the inverse measurement uncertainty propagation by Bayesian inference, based on prior available information regarding the usual dispersion of values obtained by the calibration process. The measurement uncertainties obtained using these two methods can be compared with previous results. Other relevant issues concerning this research are the broad application to measurements that require hygrometric conditions obtained from two-pressure humidity generators and, also, the ability to provide a solution that can be applied to similar iterative models. The research also studied the factors influencing both the use of the Monte Carlo method (such as the seed value and the convergence parameter) and the inverse

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uncertainty propagation using Bayesian inference (such as the pre-assigned tolerance, prior estimate, and standard deviation) in terms of their accuracy and adequacy.

Keywords Dew-point temperature · Iterative model · Measurement uncertainty

1 Introduction

The measurement of the dew-point temperature can be made either directly by a condensation hygrometer or indirectly using hygrometric instrumentation that measures input quantities such as the temperature and pressure and uses explicit mathematical models. Traceability is provided by the calibration process, establishing the link to the SI and characterizing the instrumentation accuracy level through the stated measurement uncertainty.

Metrology laboratories can provide the connection between the top levels—national metrology institutes (NMIs) and the BIPM—and lower level laboratories in industry and, therefore, have an important role in the establishment of the hygrometric traceability chains. In some cases, highly accurate reference standards are already available to be used by secondary metrology laboratories that are able to generate physical conditions according to the primary definition of the relevant quantities with low measurement uncertainties, as in the case of hygrometric instrumentation.

Some of these standards, however, are based on complex mathematical models, and require that the method applied to the evaluation of measurement uncertainty should be fit to purpose. Knowing that the international framework established by the publication of the GUM [1] in 1995 is especially suited to linear models while its Supplement 1 [2] is more able to adequately handle non-linear models, the issue of the choice of methodology is relevant for metrologists.

Nowadays, humidity generators are becoming important reference hygrometric standards in many metrology laboratories as they provide stable and uniform test conditions. In the case of a two-pressure humidity generator, the dew-point measurement is obtained indirectly from pressure and temperature measurements and an iterative mathematical model. The particular nature of this model constitutes a challenge in terms of measurement uncertainty evaluation.

Some authors, such as [3], have adopted the GUM method to obtain a solution for this specific problem. An alternative Monte Carlo approach has been proposed [4] as more suitable to deal with the non-linearity of the mathematical model. The results obtained from both approaches are very close and show only minor differences. However, the above-mentioned Monte Carlo approach [4] involves a simplified numerical method for the determination of the dew-point temperature and does not take into account the uncertainty related to the saturator efficiency. In this study, this influence factor is included in the evaluation.

The two alternative approaches proposed in this study were based on GUM Supplement 1 [2]: forward measurement uncertainty propagation by the Monte Carlo method (MCM) and the inverse measurement uncertainty propagation method by Bayesian inference (BI).

The Monte Carlo approach consists of the direct propagation of the measurement uncertainties identified for the input measurement quantities to the dew-point temperature following a numerical procedure adapted to the use of an iterative mathematical model. A detailed list of input measurement uncertainties was established for the two-pressure humidity generator under study. This humidity generator uses the Newton–Raphson method to obtain the dew-point temperature estimate.

The BI approach implemented here consists of a simplified numerical process that overcomes the implicit nature of the applied mathematical model. Two stages can be identified in this process. The first stage gives the probabilistic distribution of the observed water vapor pressure in the test chamber. This is accomplished through the use of the MCM forward procedure. The second stage uses this measurement and combines it with prior information of the dew-point temperature calibration data (usually the calibration laboratory knows the typical dispersion of values related to a certain hygrometer) to perform inverse measurement uncertainty propagation by BI.

Considering the use of the two-pressure humidity generator as a dew-point temperature reference standard, this study aims to increase the knowledge about its performance, especially discussing the use of the two numerical methods mentioned above to evaluate the measurement uncertainty and compare it with results from previous calculations [3,4].

2 Two-Pressure Humidity Generator Iterative Model

The two-pressure humidity generator operation principle consists of air saturation with water vapor at high pressure followed by an isothermal expansion to lower pressure, usually to atmospheric pressure. Both the relative humidity and the dew-point temperature are indirectly obtained by measuring the pressure and temperature at the two main components of the generator: the saturator and the test chamber.

Assuming that the condensation phenomenon does not occur during the expansion process (the expansion valve temperature is usually controlled to avoid this situation), the water vapor pressure, p_w, in the test chamber is given by

$$p_w = p_{ws}(t_d)\, f_{ws}(p_c, t_d) = \frac{p_c}{p_s} p_{ws}(t_s) f_{ws}(p_s, t_s)\eta, \tag{1}$$

where t_s and p_s are, respectively, the saturator temperature (°C) and pressure (Pa), p_c corresponds to the test chamber pressure, t_d is the dew-point temperature, p_{ws} is the saturation vapor pressure, f_{ws} is the enhancement factor, and η is the saturator's efficiency.

Both the functions p_{ws} and f_{ws} are widely studied and several alternative mathematical models can be found in the literature [5]. For this study, the Wexler [6] and Greenspan [7] models are used, including the related measurement uncertainties.

Although the best estimate of the saturator's efficiency is considered equal to one, the corresponding measurement uncertainty cannot be neglected since it is possible that the air coming out of the saturator is not totally saturated by water vapor or, on the other hand, its temperature may be greater than the saturator temperature due to insufficient cooling from the pre-saturator to the saturator.

The evaluation of the dew-point temperature using a two-pressure humidity generator requires a solution for Eq. 1 through an iterative method. The humidity generator under study applies the Newton–Raphson numerical method; other methods could be used [4].

To apply the Newton–Raphson method [8] an initial value (seed) for the dew-point temperature, t_{d_0}, is defined and n iterations are performed until the algorithm converges to a solution (within a preassigned convergence tolerance) or a limit of iterations is reached. The iterative step is given by

$$t_{\mathrm{d}_{n+1}} = t_{\mathrm{d}_n} - \frac{g\left(t_{\mathrm{d}_n}\right)}{g'\left(t_{\mathrm{d}_n}\right)}, \tag{2}$$

where

$$g\left(t_{\mathrm{d}_n}\right) = \frac{p_{\mathrm{ws}}(t_{\mathrm{s}}) f_{\mathrm{ws}}(p_{\mathrm{s}}, t_{\mathrm{s}}) p_{\mathrm{c}}}{p_{\mathrm{ws}}(t_{\mathrm{d}_n}) f_{\mathrm{ws}}(p_{\mathrm{c}}, t_{\mathrm{d}_n}) p_{\mathrm{s}}}\eta - 1, \tag{3}$$

and $g'\left(t_{\mathrm{d}_n}\right)$ is the first derivative of $g(t_{\mathrm{d}_n})$ with respect to t_{d_n}.

3 Measurement Uncertainty Evaluation

3.1 Introduction

Several approaches can be used to evaluate the measurement uncertainty. The selection of the method must consider the nature and complexity of the mathematical model combined with the effort required for its implementation and the ability of the method to provide adequate solutions. Table 1 presents some advantages and constraints of the most common applied methods [9].

Considering the implicit mathematical model nature of the studied problem (dew-point temperature measurement in the two-pressure humidity generator), both the Monte Carlo and the Bayesian inference methods can be used to evaluate the measurement uncertainty. Therefore, two procedures were proposed: the forward measurement uncertainty propagation by the MCM, adapted to the use of an iterative mathematical model by the Newton–Raphson numerical method (Sect. 3.3); and the inverse measurement uncertainty propagation (Sect. 3.4), also using the MCM, but following a simplified numerical Bayesian approach by defining a probabilistic model for the measurement and using it to update prior available information about the measurand.

3.2 Input Data and Probabilistic Formulation

In order to implement the proposed approaches presented above, it is necessary to provide experimental data for the input estimates, taking into account the studied hygrometric conditions presented in Table 2.

Table 3 presents the measurement uncertainties related to the input quantities, namely, the source of uncertainty, adopted probability distribution functions (PDFs),

Table 1 Measurement uncertainty evaluation methods

Method	Advantages	Constraints
Analytical	Provides "exact" solutions for the propagation of measurement uncertainties from input quantities to output quantities considering both linear and non-linear mathematical models	The convolution process generally increases the complexity of the calculus
Mainstream GUM	Generally provides adequate solutions for measurement uncertainty propagation for linear or linearizable models	When applied to strongly non-linear mathematical models, it gives approximate solutions that can be highly inaccurate. It requires the use of symmetrical probability distribution functions
Monte Carlo	Numerical method that allows approximate solutions that converge to the "exact" solutions. Allows a good probabilistic formulation of the input quantities since both symmetrical and non-symmetrical probability density functions can be used	The knowledge about physical limits of the measurand is treated in a functional way. The obtained results may differ from the ones originated by alternative approaches such as Bayesian inference
Bayesian inference	It considers prior information of the measurand, including physical constraints	Can be dependent on the quality of the available prior information

Table 2 Input data for the studied hygrometric conditions

Temperature (°C)	Dew-point temperature (°C)	Relative humidity (%)	Saturator pressure (kPa)	Saturator temperature (°C)	Test chamber pressure (kPa)
20	1.92	30	339.3	19.99	101.3
	9.30	50	202.5	19.99	101.3
	19.24	95	106.2	20.00	101.4

and typical values for the standard uncertainties usually associated with the humidity generator under consideration.

3.3 Forward Measurement Uncertainty Propagation by the Monte Carlo Method

The forward measurement uncertainty propagation process requires MCM simulation according to the guidelines presented in the GUM Supplement 1 [2]. The computational process implies the generation of a set of numerical sequences of input quantities (test chamber pressure, saturator temperature, pressure, efficiency, saturation vapor pressure, and enhancement factor functions), i.e., the individual pseudo-random generated numbers combined with a dew-point temperature seed value to perform an iterative process based on Eqs. 2 and 3.

Table 3 Measurement uncertainty of the input quantities

Source of uncertainty	PDF	Standard uncertainty
Saturator temperature, t_s		
Calibration	Gaussian	0.007 5 °C
Drift	Triangular	0.006 1 °C
Self-heating	Uniform	0.005 8 °C
Resolution	Uniform	0.002 9 °C
Homogeneity	Uniform	0.012 °C
Stability	Gaussian	0.002 °C
Repeatability	Gaussian	0.005 °C
Saturator pressure, p_s		
Calibration	Gaussian	130 Pa
Drift	Triangular	112 Pa
Resolution	Uniform	2 Pa
Internal pressure difference	Uniform	100 Pa
Stability	Gaussian	60 Pa
Pressure transducer location	Uniform	12 Pa
Test chamber pressure, p_c		
Calibration	Gaussian	130 Pa
Drift	Triangular	90 Pa
Resolution	Uniform	2 Pa
Stability	Gaussian	20 Pa
Reversibility or hysteresis	Uniform	69 Pa
Saturation vapor pressure, p_{ws}	Uniform	0.002 9 %
Enhancement factor function, f_{ws}	Uniform	0.002 9 %
Saturator efficiency, η	Triangular	0.001 4

The output dew-point temperature obtained at the end of the iterative process is then considered an element of the numerical sequence representing the output quantity. A representation of this process can be seen in Fig. 1.

The seed and convergence parameters related to the iterative process should be considered as possible influence quantities to the output results and, consequently, a parametric study of those effects must be carried out. With this purpose, several seed and convergence values were applied and the corresponding output results were compared.

It must be emphasized that the accuracy of the solutions obtained through this approach is strongly dependent on the quality of the tools used to perform the computational calculations. Our studies used validated tools such as the Mersenne-Twister pseudo-random number generator [10], known PDF sequence converters, and an optimized sorting algorithm [11]. The obtained individual sequences were composed of 10^6 elements and the computational accuracy level of the numerical simulations was achieved using the methodology described by Cox et al. [12], setting a maximum computational accuracy level of ±0.005 °C, for a 95 % confidence interval.

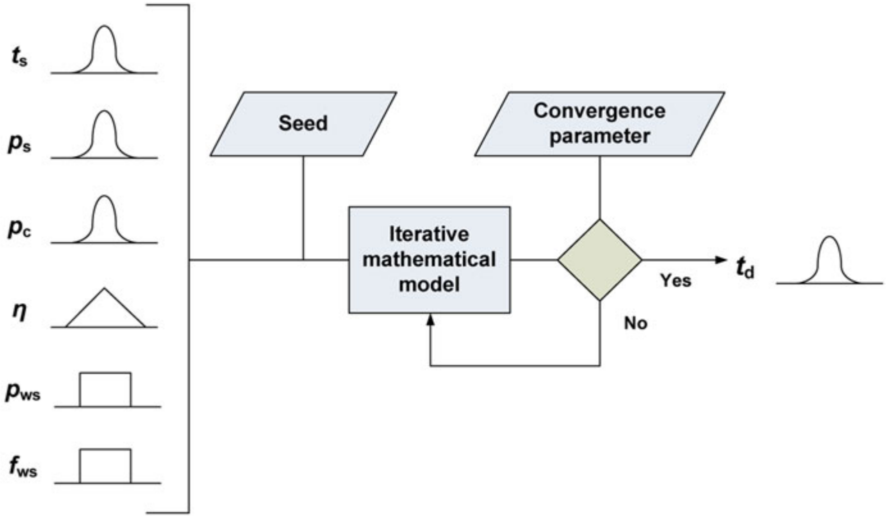

Fig. 1 Forward measurement uncertainty propagation

3.4 Inverse Measurement Uncertainty Propagation by Bayesian Inference

The proposed approach, named inverse measurement uncertainty propagation [13], is based on Bayesian inference since it provides a way to obtain a numerical sample from an approximation to the dew-point temperature posterior PDF, $p(t_\mathrm{d}|p_\mathrm{ws})$, based on the specification of a prior PDF, $p(t_\mathrm{d})$, expressing the existing probabilistic knowledge about the measurand before the calibration test (supported by previous calibration results of the same hygrometer), and a likelihood, $p(p_\mathrm{ws}|t_\mathrm{d})$, representing the probability of observing p_ws if the true dew-point temperature is t_d. In this case, Bayes' theorem can be stated as

$$p(t_\mathrm{d}|p_\mathrm{ws}) \propto p(p_\mathrm{ws}|t_\mathrm{d})\, p(t_\mathrm{d}). \tag{4}$$

This approach makes use of the MCM in two stages (see Fig. 2). In the first stage, an MCM forward measurement uncertainty can be used to provide the numerical sequence representing the observed water vapor pressure, p_w, in the test chamber. In this case, the MCM is used to characterize the likelihood $p(p_\mathrm{ws}|t_\mathrm{d})$. In the second stage, the MCM allows generation of numerical samples of dew-point temperatures, $t_{\mathrm{d},q}$, from $p(t_\mathrm{d})$ and then saturation vapor pressure values, $p_{\mathrm{ws},q}$, from $p(p_\mathrm{ws}|t_{\mathrm{d},q})$. At this stage the pairs $(t_{\mathrm{d},q}, p_{\mathrm{ws},q})$ represent samples from the joint distribution $p(p_\mathrm{ws}, t_\mathrm{d}) = p(p_\mathrm{ws}|t_\mathrm{d})p(t_\mathrm{d})$. A sample from the posterior distribution $p(t_\mathrm{d}|p_\mathrm{ws})$ is generated by choosing those samples $t_{\mathrm{d},q}$ for which the corresponding $p_{\mathrm{ws},q}$ is close to the observed p_w, i.e., for some tolerance τ,

$$\left\{t_{\mathrm{d},q} : \left|p_{\mathrm{ws},q} - p_\mathrm{w}\right| < \tau\right\}. \tag{5}$$

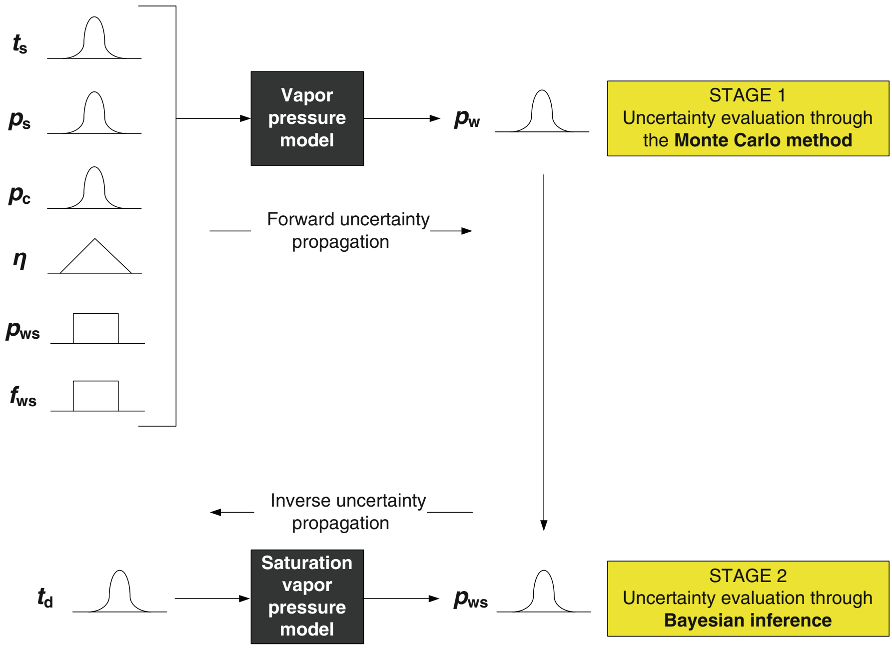

Fig. 2 Forward and inverse measurement uncertainty propagation

The tolerance parameter mentioned above has an important role in this second stage because of its direct relation to the output sequence, namely, its dimension (number of elements) and computational accuracy level. Due to the relevance of this assigned condition, several reasonable values were applied to quantify their effect on the final results.

The prior probabilistic knowledge about the measurand (estimate and measurement uncertainty) is also an assigned condition influencing the output results. With the purpose of evaluating its influence and assuming that a Gaussian PDF was adopted to describe the prior state of knowledge of the dew-point temperature quantity, two sets of values of prior estimates (9.10 °C, 9.30 °C, and 9.50 °C) and standard deviations (0.05 °C, 0.1 °C, 0.5 °C, 1.0 °C, and 1.5 °C) were applied.

The maximum computational accuracy level of 0.005 °C (with a 95 % confidence interval) was taken from Sect. 3.3 for the posterior PDF numerical sample.

4 Results

4.1 Forward Measurement Uncertainty Propagation

The MCM approach was applied to several hygrometric conditions: relative humidity of 30 %, 50 %, and 95 %, considering a temperature of 20 °C. Table 4 presents the output results of the dew-point temperature estimates, expanded measurement

Table 4 Forward measurement uncertainty approach output results

Dew-point temperature estimate (°C)	Dew-point temperature measurement uncertainty (95 % confidence interval) (°C)	Computational accuracy level (95 % confidence interval) (°C)
1.935	0.069	0.001 2
9.315	0.077	0.001 3
19.257	0.098	0.001 6

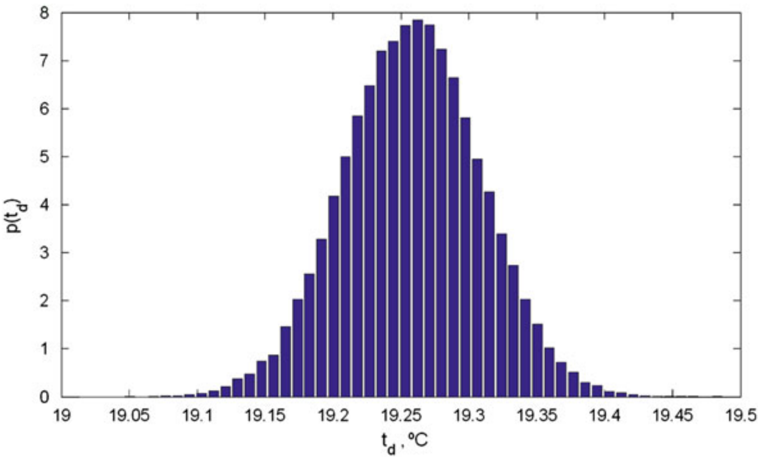

Fig. 3 Output PDF for a reference condition of 20 °C and 95 %rh

uncertainties, and computational accuracy levels (for 95 % confidence interval). A seed value of 10 °C and a convergence parameter of 0.001 °C were considered.

The obtained results show that the expanded measurement uncertainty changes between ±0.069 °C and ±0.098 °C for the measuring interval of 1.935 °C to 19.257 °C. The related computational accuracy level is less than ±0.002 °C in all cases and complies with the pre-defined maximum value of 0.005 °C.

Figure 3 presents the PDF of the dew-point temperature quantity for a reference condition of 20 °C and 95 %rh. Its shape is close to that of a Gaussian PDF, as expected under the central limit theorem.

A comparison with previous results shows that the measurement uncertainties obtained by the proposed approach produce a higher magnitude. In fact, the application of the GUM method [3] revealed that expanded measurement uncertainties are between 0.034 °C and 0.040 °C for a dew-point temperature measuring range of 0.66 °C to 25 °C. The use of the MCM described in [4], without considering the use of the Newton–Raphson method or the saturator's efficiency, provided measurement uncertainty values from 0.032 °C to 0.035 °C for dew-point temperature estimates between 10 °C and 95 °C.

In order to study the reasons for these differences, a GUM approach [1] was implemented (using the complex-step method [14,15] to evaluate all partial derivatives)

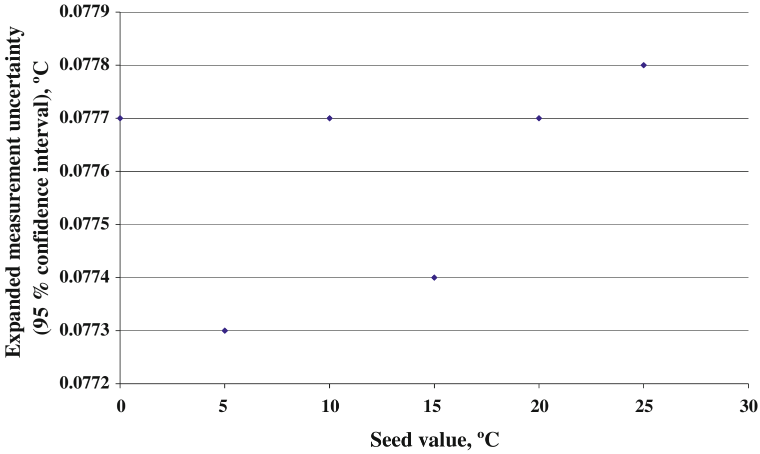

Fig. 4 Relation between seed parameter and output simulation results

based on the same input probabilistic information presented in Table 3. The obtained results are very close to the measurement uncertainties presented in Table 4, produced by the forward MCM approach. This fact confirms that for this example, when using the same input data, both the GUM and the MCM approaches give similar results. The non-linearity of the underlying mathematical model is not sufficiently strong to invalidate the results obtained by the GUM approach.

The study of the relation between the seed and convergence parameters with the output results given by the forward measurement uncertainty approach was made for the hygrometric condition of 20 °C and 50 %rh. In the first case (seed parameter versus output results), the convergence parameter was constant, equal to 0.001 °C, while in the second case (convergence parameter versus output results), a seed value of 10 °C was used. The results are shown in Figs. 4 and 5, respectively, with the dew-point temperature output estimate equal to 9.30 °C for all the studied cases. The computational accuracy levels of the output numerical simulations were approximately identical and again less than 0.002 °C.

Figure 4 shows that the seed parameter has a small influence and random behavior on the measurement uncertainty, with a maximum observable variation of 0.000 5 °C (considering the set of seed values).

From Fig. 5 it is possible to observe that the measurement uncertainty decreases when considering tighter convergence parameters, i.e., close to 0.000 05 °C. Although the variations have a magnitude close to the computational accuracy level of the numerical simulations, this systematic effect cannot be neglected.

4.2 Inverse Measurement Uncertainty Propagation

The Bayesian approach was implemented for a hygrometric reference condition of 50 %rh, considering a temperature equal to 20 °C and assuming a dew-point temperature Gaussian prior distribution centered at 9.3 °C with a standard deviation of 0.5 °C,

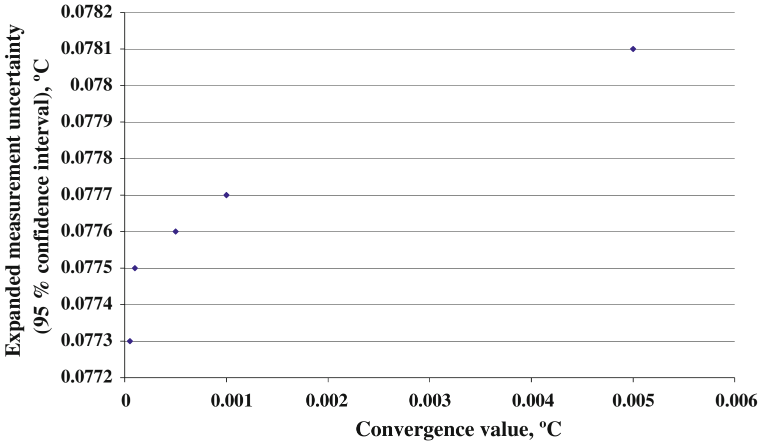

Fig. 5 Relation between convergence parameter and output simulation results

Table 5 Influence of the preassigned tolerance parameter on the output simulation results

Tolerance (Pa)	Dew-point temperature estimate (°C)	Dew-point temperature measurement uncertainty (95 % confidence interval) (°C)	Computational accuracy level (95 % confidence interval) (°C)	Dimension of the output numerical sequence
130	9.30	0.98	0.005 6	998 805
100	9.30	0.94	0.004 7	988 048
70	9.29	0.77	0.002 6	922 227
50	9.29	0.58	0.001 6	792 712
40	9.29	0.47	0.001 2	685 966
30	9.29	0.36	0.001 1	550 373
20	9.30	0.25	0.001 0	385 553
10	9.30	0.14	0.001 1	199 225
5	9.30	0.091	0.001 5	100 041
1	9.30	0.062	0.002 2	20 133
0.5	9.30	0.060	0.003 4	10 290
0.1	9.30	0.062	0.006 5	1 979

which represents the available laboratory knowledge about the measurand. Several values of the preassigned tolerance parameter were applied in the performed numerical simulations to determine its influence on the output sequence. The obtained results are presented in Table 5.

Although the dew-point temperature estimate remains indifferent to the influence of the preassigned tolerance, both the corresponding measurement uncertainty and the dimension (number of elements) of the output numerical sequence change when using different values for this input parameter. The use of a smaller preassigned tolerance

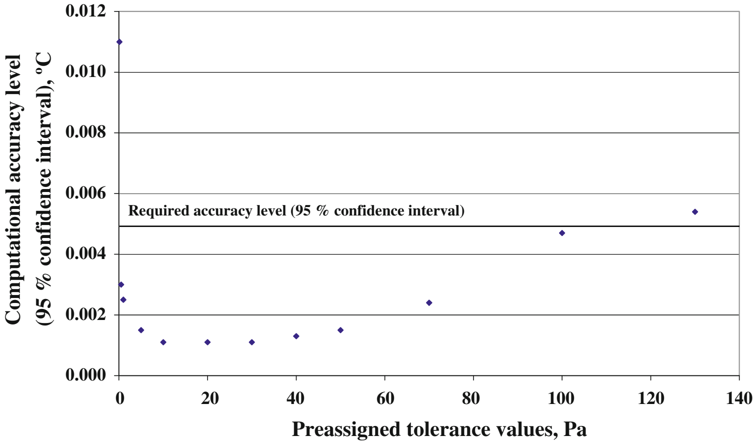

Fig. 6 Effect of the preassigned tolerance on the computational accuracy level

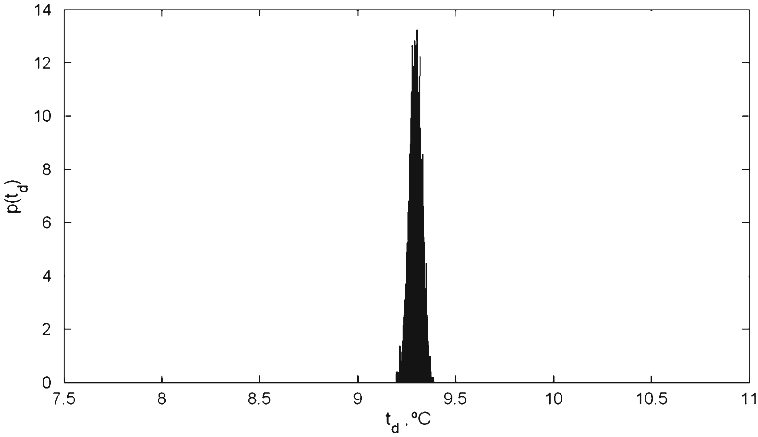

Fig. 7 Posterior PDF for a 0.1 Pa preassigned tolerance

value gives a better numerical approximation to the posterior PDF (closer to the posterior PDF eventually obtained through an analytical procedure), but leads to a smaller sample and, hence, a less valid inference. Conversely, setting a larger tolerance yields a higher output sequence dimension and, therefore, a more valid inference, although a less accurate approximation to the posterior PDF. A balance can be made through the obtained computational accuracy level related to the output numerical sequence, represented in Fig. 6, where a minimum value is found for a tolerance close to 20 Pa.

The use of the MCM allows visualization of the influence of the preassigned tolerance parameter on the spread and shape of the output PDFs (Figs. 7, 8, 9).

From the PDFs presented above, it is possible to consider that the use of higher tolerances leads to an output PDF with a Gaussian shape.

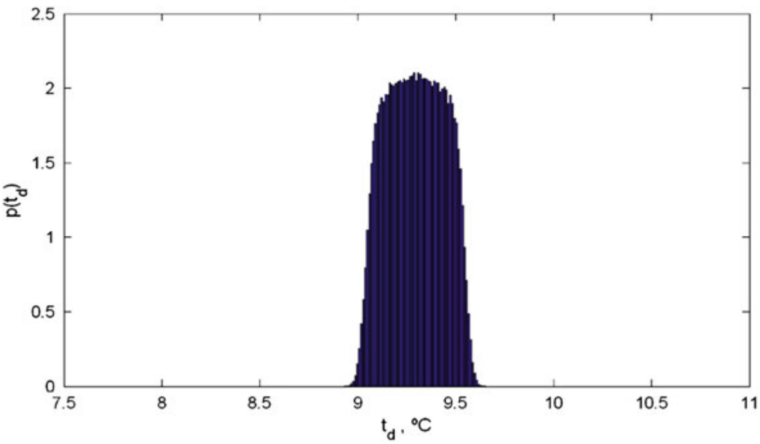

Fig. 8 Posterior PDF for a 20 Pa preassigned tolerance

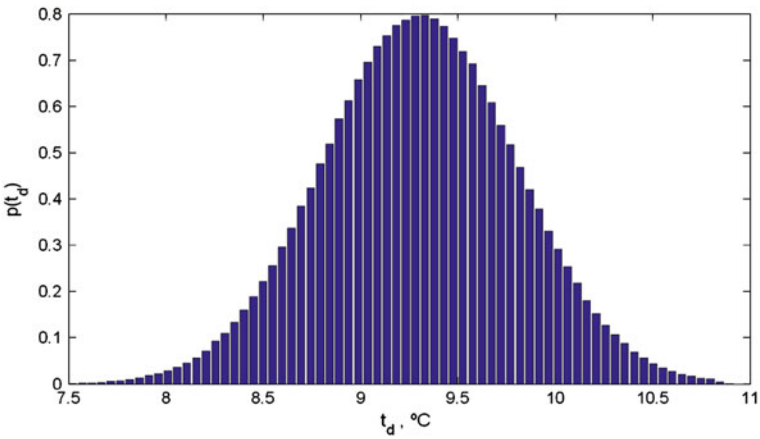

Fig. 9 Posterior PDF for a 130 Pa preassigned tolerance

The study of the influence of prior knowledge about the measurand, which included both the prior estimate and the standard deviation of the adopted Gaussian PDF, was made considering the preassigned tolerance corresponding to the best computational accuracy level achieved (in the present case 20 Pa) for the output numerical simulation.

In the first case, several prior dew-point temperature estimated values were tested in the interval of 9.10 °C to 9.50 °C. The obtained results show that both the measurement uncertainty and the computational accuracy levels remain approximately constant, 0.25 °C and 0.001 °C, respectively. However, a small shift in the output temperature estimate is noticed. As the prior dew-point estimate was increased from 9.10 °C to 9.50 °C, while maintaining the prior standard deviation constant and equal to 0.5 °C, the output dew-point estimate increased from 9.28 °C to 9.31 °C.

In the second case, a prior dew-point temperature estimate of 9.3 °C was assumed and several prior standard deviations values from 0.05 °C to 1.5 °C were applied in

Table 6 Influence of the prior dew-point temperature standard deviation in the output results

Prior dew-point temperature standard deviation (°C)	Dew-point temperature estimate (°C)	Dew-point temperature measurement uncertainty (95 % confidence interval) (°C)	Computational accuracy level (95 % confidence interval) (°C)	Dimension of the numerical sequence (°C)
0.05	9.30	0.10	0.000 6	999 964
0.1	9.30	0.19	0.000 8	983 920
0.5	9.30	0.25	0.001 0	385 218
1.0	9.29	0.25	0.001 5	198 873
1.5	9.30	0.25	0.001 8	133 487

the performed numerical simulations. The simulation output results are presented in Table 6.

Although the obtained estimate remains constant for the several standard deviation values tested, it is possible to observe an increase of the measurement uncertainty from $\pm 0.10\,°\mathrm{C}$ to $\pm 0.25\,°\mathrm{C}$ when considering higher standard deviations values, and hence, weak prior knowledge about the measurand. It is also possible to observe that a higher prior standard deviation give an output sequence with a lower number of elements and thus a worst computational accuracy level is achieved, making the approach less efficient.

5 Conclusions and Future Developments

This article shows how two different approaches—forward and inverse measurement uncertainty propagation—can be used to evaluate the measurement uncertainty of the dew-point temperature indirectly measured by a two-pressure humidity generator through the use of an iterative mathematical model.

The MCM forward approach produced measurement uncertainties higher than the previous known values obtained by the use of the GUM method [3] or the MCM in [4]. In order to make a reliable comparison, a GUM approach was implemented, using the same input data as the MCM forward approach. The results confirmed the magnitude of the obtained measurement uncertainties, revealing that the GUM approach is sufficiently accurate to perform a measurement uncertainty evaluation for the two-pressure humidity generator and, therefore, should be preferred due to its simplicity.

Regarding the influence factors of the MCM forward approach, this study indicates that the seed value has limited influence on the obtained dew-point temperature measurement uncertainty. However, the convergence parameter has a clear influence and a tight value should be used to reduce its impact on the final results.

The BI inverse approach avoids the iterative process by assuming a probabilistic model for the dew-point measurement used to update prior information about the measurand and thus obtaining the posterior PDF, in the present case, by a simplified numerical method based on Monte Carlo simulations. The preassigned tolerance, used in the implementation of the proposed numerical approach, has a major influence on both the obtained measurement uncertainties and the computational accuracy level. Other influence factors found are the prior estimate and standard deviation which can change the output estimate and related measurement uncertainty.

These limitations make the simplified BI inverse approach much less robust, obtaining results that differ significantly from the other two studied approaches (MCM and GUM). The potential advantage of the simplified BI approach is that it can be implemented by sampling from standard distributions (as does the MCM). The disadvantages are that it is computationally expensive in that nearly all generated samples are thrown away and secondly, the validity of the posterior distribution depends on the tolerance parameter.

Due to these limitations, future developments will be focused on alternative calculation approaches to implementing a Bayesian approach. Alternative and more advanced numerical procedures should be studied (i.e., Monte Carlo sampling methods based on Markov chains), making possible the use of a more efficient Bayesian approach and comparing it with the results presented in this study. However, it has already been mentioned that the nonlinearities in the underlying model do not have a large effect on the evaluated uncertainties. In this case, the Bayesian posterior distribution is likely to be similar to that derived using the GUM and MCM approaches.

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